An Index of Dynamic Density

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The risk of a collision between aircraft is rising at the density of commercial air traffic increases. This trend, together with the overwhelming need to upgrade the National Airspace System, has motivated the Federal Aviation Administration to sponsor the development of metrics to evaluate "dynamic density" – a proxy for the likelihood of collision risk. Here we propose and evaluate a mathematical index of dynamic density, D, that describes collision risk. Although our domain of investigation is aviation, the logic of D is applicable whenever objects move in limited spaces. A series of sensitivity analyses illustrate how D responds to frequently encountered air traffic conflict situations. We illustrate a use of D that characterizes pilot performance and efficiency in experimental simulations of free flight and suggest other human factors applications. This research could be applied immediately by the traffic management units of en-route air traffic control centers to reformulate the criterion for the critical capacity of sectors.

THE DESIGN PROBLEM

Increases in air traffic will continue to limit system capacity and drain valuable organizational resources and capital. To counter these structural problems, the Federal Aviation Administration (FAA) and the civil aviation community are committed to redesigning the National Airspace System (NAS) in order to improve the accessibility, flexibility, and safety of air transportation.

A federal advisory committee, the Radio Technical Commission for Aeronautics (RTCA) Task Force 3, has recommended a transition from today's rigid, procedural, analog, ground-based system to a flexible, collaborative system (RTCA, 1995). It has established the goal of creating a seamless air traffic management system by the year 2010 that takes advantage of the revolution in communication, navigation, and surveillance technologies (e.g., universal two-way data link, satellite-based navigation and surveillance).

Free flight is a system concept that addresses this goal. In the concept of free flight, it is proposed that the responsibility and authority for decisions concerning routing and separation be redistributed from centralized ground-based air traffic control (ATC) to the flight decks of commercial airlines. This redesign of the NAS has been dubbed "free flight" because it removes constraints on flight planning and emphasizes flight deck management, especially en-route separation (FAA, 1995).

The primary difference between the current structure of the NAS and free flight is a relaxation of constraints on clearances for routing, speed, and altitude. Imposition of constraints by ground-based controllers will be necessary only when potential maneuvers might interfere with other aircraft, when traffic density precludes free-flight operations, when unauthorized entry of special-use airspace is imminent, or when flight restrictions are considered necessary for safety (RTCA, 1995). In a free-flight environment, each aircraft will fly a dynamic, user-preferred flight path, making full use of on-board communication, navigation, and surveillance systems. The proposed role of ground-based air traffic managers in free flight will be separation monitoring and prediction.

The human factors issues raised by the
transition from a centralized to a distributed decision-making system are those raised by all "wicked design problems" (Rittel & Webber, 1973; Vicente, Burns, & Pawlik, 1997). For such designs the initial task of formulating the problem can be a considerable undertaking. Subsequent tasks, such as planning for contingencies that are not yet conceived, addressing the iterative and nonlinear nature of large system development, and accommodating the contrasting viewpoints of multiple stakeholders, are themselves ill-defined design problems. Compounding this "wickedness" is the demand that the development of concepts, procedures, and technologies must continue to address the fundamental constraint is the NAS safety. All efforts must continue to respond to the challenge of making timely, accurate, efficient, and safe decisions.

One operational concept developed by the RTCA task force to guide these efforts is dynamic density. This is an observer construct that is open for formulation. The task force indicated that any proposed formulation of dynamic density should reflect the impact of essential factors affecting separation and conflict in both en-route and terminal airspace. Among these factors are the number of aircraft and the complexity of flow of aircraft (comprising aspects of spatial relations and dynamic trajectories). Thus dynamic density is viewed as more than a count of aircraft per unit volume. The RTCA has called for the development of a dynamic density concept and an accompanying metric that can be used to assess complexity, projected workload, and the risk of collision posed by traffic in an operational environment. Measuring and predicting changes in dynamic density are viewed as necessary steps for managing the unstructured traffic flows envisioned in the free-flight environment.

The RTCA has identified the development of a dynamic density metric as a "high-priority item" (RTCA, 1995, p. 33). Furthermore, for the metric to provide adaptability and flexibility in the decision support system, the RTCA anticipates that the metric must be sensitive to separation standards, traffic demand, airspace configurations (spatial relationships), altitude profiles, and levels of threat. The metric of dynamic density that will ultimately be adopt-
ed by the aviation community will serve as a yardstick for the likelihood of self-separation violations — that is, as an index of the risk of collision in free flight.

In this paper we present a model of dynamic density that provides an initial approach to the assessment of the complexity of spatial dynamics that focuses on separation, which is the most important factor in the mitigation of collision risk. We define an index of dynamic density (D) that is a component-based function of time and of the number and distribution of objects (aircraft) in a sector of (air)space. Large values of D represent high density; values near zero represent a traffic situation with little likelihood of collision. Changing values of D across time represent the dynamic density of traffic.

We propose that the dynamic density metric can also be employed to solve human factors problems. Patterns of dynamic density values — especially those associated with loss of separation — provide unique insight into relationships between performance (e.g., pilot avoidance maneuvers) and changing airspace configurations.

THE INDEX OF DYNAMIC DENSITY

The equation used to define the index of dynamic density, D, is:

$$D = \frac{1}{\left(\frac{d}{a}\right)^c}$$  \hspace{1cm} (1)$$

where d is the distance between two aircraft at the same altitude, c is a normalization constant, and a is a weighting parameter. The index considers only aircraft near the same altitude because aircraft must be at the same altitude in order to collide. We adopt an operational definition of "near the same altitude" from current FAA standards: within ±1000 feet when below 29,000 feet (FL290), and within ±2000 feet above FL290. Aircraft separated by several thousand feet vertically are not treated as a hazard here because they do not contribute to our definition of airspace density at this time. This formulation extends to in-
craft that are climbing or descending, as well as to those in level flight.

The normalization constant, c, determines the lateral separation at which the ratio (d/c) exceeds 1. We typically set c to 5 nautical miles (nm; 9.27 km), which is the minimum allowable lateral separation by current FAA regulations. With c set to 5 nm (9.27 km), it embodies the industry standard definition of the separation at which traffic density becomes unacceptable. The value of the index equals 1 when aircraft breach the 5 nm (9.27 km) performance criterion. As aircraft get closer, the ratio (d/c) approaches 0 and the value of D increases rapidly without bound. Thus large values of D indicate high traffic density. As long as aircraft remain separated, the ratio (d/c) exceeds 1 and the value of D remains relatively small.

One implication of this formulation is that two aircraft in close proximity create a condition of “dense” traffic. This implication coheres to everyday experience—traffic is always dense prior to any collision (or near miss).

The weighting parameter, a, preferentially weights the contribution of “close” aircraft (where d < c) and suppresses the relative contribution of more distant aircraft. Larger values of a cause the value of D to increase more quickly when the ratio (d/c) is less than 1. The selection of a value for a is an empirical issue. The goal is to find a reasonable balance between the relative contributions of close and distant aircraft. In our application a comparison of self-separation scenarios with as many as 16 aircraft and up to three simultaneous conflicts, we have found that setting a = 3 meets our needs. The reader is, however, encouraged to experiment with the parameters in the equation.

THE SINGLE AIRCRAFT PAIR CONDITION

Figure 1 illustrates an en-route flight scenario in which a single pair of aircraft approach head-on, missing each other by 5 nm (9.27 km)—a situation that might occur in a free-flight environment in which flight decks are ceded the authority to make decisions about routing and separation. Figure 2 illustrates the behavior of Equation 1 with a = 3 and c = 5 for the scenario of Figure 1. In Figure 2, the

value of D is plotted over two horizontal axes representing time and distance. The relationship between Figures 1 and 2 illustrates the use of D.

In Figure 2, Segment A has a positive slope. The positive slope marks the time when relatively distant aircraft are moving closer together. The peak value of D occurs at the time of minimum separation. The negative slope (Segment B) indicates that aircraft are moving further apart. The rate of change of the value of D across time discriminates between approaching and receding aircraft—that is, between increasing and decreasing levels of dynamic density.

Crossing and Merging Aircraft

Figure 3 illustrates a pair of common intersection situations: crossing aircraft (symbolized as X) and aircraft converging to a single path (symbolized as Y). Figure 3a shows a crossing situation in which Aircraft T2 is slightly faster and reaches the intersection point slightly earlier than Aircraft T1. The crossing situation is characterized by a rapid decrease in separation followed by an equally rapid increase in separation. Figure 3b shows a converging-to-path situation with T2 again slightly faster than T1. T1 occupies the lead position after convergence. This case is characterized by a rapid decrease in separation followed by a slow, steady decrease in separation.
caused by the slightly higher speed of the trailing aircraft, T2.

The resulting time plots of D are shown in Figure 4. The dynamic density for the crossing scenario of Figure 3a is plotted in Figure 4a, and the dynamic density for the converging-to-path scenario of Figure 3b is plotted in Figure 4b. The shape of the curve in Figure 4a is symmetrical because the speeds and directions of aircraft are constant. Crossing traffic produces a rapid increase in dynamic density followed by an equally rapid decrease in density. In contrast, Figure 4b is not symmetrical because T1 changes direction. Converging-to-path produces a rapid increase in dynamic density, followed by one of three alternative conditions. The conditions are determined by the relative velocity of the aircraft in trail. The three lines that form the right-hand tail of Figure 4b indicate the portion of flight after converging to path under the three conditions. In each case, T2 is in-trail behind T1.

Condition 1 represents the situation of Figure 3b, in which T1 and T2 are slowly closing after converging to path. The gentle rise in the value of D indicates a slow, steady increase in dynamic density as T2 overtakes T1.

Condition 2 represents an alternative situation in which T1 and T2 maintain constant separation after converging to path. The constant value of D indicates a constant level of dynamic density.

Condition 3 represents the situation in which T1 and T2 slowly diverge after converging to path. The negative slope signals decreasing density.

Figure 3. (a) The X crossing and (b) Y converging-to-path scenarios. Angle α is the angle of approach.
Sensitivity to Relative Velocity

The left-hand, upward-sloping segment of the curve in Figure 4b results from Aircraft T1 and T2 approaching each other at angle α. The angle of approach (α) affects the rate of closure – that is, the relative velocity between T1 and T2. As shown in Figure 5, the index is sensitive to changes in relative velocity between aircraft. Given constant aircraft speeds, relative velocity increases as α approaches 180° (head-on) and the peak narrows. The width of the peak reflects the rate of closure.

It is not possible to differentiate the components of relative velocity by inspecting the value of D alone. Relative velocity is a function of three components: the angle of approach (α) and the speeds and starting positions of each aircraft. Changing any of these components changes the shape of the time plot of D. For example, increasing the speed of either aircraft increases the slope and narrows the peak of the curve.

MULTIPLE AIRCRAFT PAIRS

Equation 2 extends the index in order to capture the interaction between the pilot’s aircraft ("ownership") and multiple aircraft traffic. In this formulation the altitude of the ownership defines a so-called reference altitude. The summation takes the sum of components, with each component representing the dynamic
density associated with an aircraft (target) near the reference altitude:

\[
\sum_{j=1}^{N} \left( \frac{d_j}{c} \right)^a
\]

\[\text{(2)}\]

where \( j \) enumerates aircraft (targets) near the reference altitude.

To appreciate the utility of this formulation, one might consider the example shown in Figure 6, in which an ownership encounters four approaching targets. In this case, the pilot of the ownership should respond to – and the index of dynamic density should reflect – the possibility of four separate potential conflicts.

The time plot of Equation 2 for the scenario of Figure 6 is given in Figure 7. The speeds of the target aircraft are assumed to be identical, and the normalization index \( c \) is set to 5 km (9.27 mi). The composite index shows four separate peaks corresponding to the times of minimum separation from each of the four target aircraft.

Figure 7 also illustrates the effect of varying the weighting function \( a \). Lower values of \( a \) blur information about the different aircraft. Such blurring of information is a problem for any composite index. With higher values of \( a \), the effect of relatively distant aircraft is more suppressed, and the primary effects of the closest pairs become evident. Increasing the value of the weighting function allows the peaks to remain distinct even when multiple aircraft converge.

An analogous expansion adds a second summation in order to consider all \( N(N-1)/2 \) pairings of aircraft at a given altitude (Equation 3). This formulation of \( D \) yields a global index of traffic density:

\[
\sum_{i=1}^{N} \sum_{j=i+1}^{N} \left( \frac{d_{ij}}{c} \right)^a
\]

\[\text{(3)}\]

**DISCUSSION**

**Comparison with Alternative Metrics**

Our proposal for the metric of dynamic density that is sought by the RTCA task force is the first of which we are aware. Our formulation intentionally simplifies the complexity of spatial dynamics in order to focus on separation within an altitude layer. We are in the process of developing further refinements to \( D \) that integrate vertical separation with lateral separation. When alternative formulations become available, it will be possible to compare them with our proposed metric both empirically and analytically. Because there can be no comparative analyses at this time, our discussion focuses on how we use \( D \) in our laboratory and in other human factors applications.

**Laboratory/Experimental Application**

The index of dynamic density can be used to interpret pilot efficiency and safety in simulated self-separation traffic scenarios. In our simulation facility, we ask commercial pilots to navigate an aircraft in a manner consistent with a preprogrammed flight plan and to progress to a destination airport (Scallen, Smith, & Hancock, 1996). The experienced, volunteer pilots were instructed that they had full authority to execute decisions for routing (course, altitude, and speed) and separation with none of the current FAA-mandated routing and altitude restrictions in effect. The only requirement was to maintain

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*Figure 6. Interaction of an ownership (O) and four targets (T1-T4).*

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a minimum separation of 5 nm laterally and 1000 feet vertically.

Cockpit instrumentation included a primary flight display, a cockpit display of traffic information, and a flight management system consisting of a display screen and a Honeywell keyboard input device designed for the Boeing 757. Each traffic scenario began with the pilot’s ownership at cruise altitude part of the way through the flight plan. In 11 of the 12 scenarios, the pilot encountered an en-route traffic situation composed of drones (aircraft that pose no potential conflict for the pilots’ path) and one or more “targets” (aircraft that will eventually conflict with the pilots’ path). The self-separation scenarios are characterized by complexity factors such as number of aircraft, traffic type, and configuration (e.g., crossing or converging traffic), and angle of approach.

The traditional criterion for pilot performance is an operational error, defined binomially as either “success” or “failure” in adhering to an arbitrary rule of separation (in our research it is 5 nm laterally and 1000 ft vertically). Aircraft pairs that violate the rule make an operational error. Although this approach provides an indication of overall performance, it does not indicate how self-separation was lost.

In contrast, graphical values of the index of dynamic density against elapsed time produces a visual presentation for the process of separation loss. Figure 8 provides an example of this. The baseline reveals the pattern of dynamic density that results from the programmed scenario (e.g., from the initial configuration of aircraft and their flight plans). Density rises rapidly at the beginning of the scenario as the target aircraft closes on the ownership. The baseline reaches a value of 1.0 as the time of the programmed loss of separation, approximately 1.5 min into the scenario. This pattern is reproduced if a pilot makes no maneuvers. Figure 8 plots experimental data for four commercial pilots in the same scenario. Plotting dynamic density in this format yields both quantitative and qualitative information. For instance, of the four pilots, only Pilot 2 (whose performance is shown by the triangular symbols) produced values exceeding the critical - i.e., 1.0 - threshold; he was the only pilot to fail to maintain separation. He lost separation (an operation error) approximately 5 min into the scenario, 1.5 min after the programmed loss of separation.

The other three pilots maintained separation, By traditional measures, their performance
would be deemed identical. However, the index of dynamic density indicates that this was not the case. The pilots made radically different maneuvers that produced diagnostically different patterns of dynamic density. For instance, the rounded inflection points and gentle slopes of the curves for Pilots 2 and 3 (shown by triangles and diamonds, respectively) indicate that these pilots exerted lateral maneuvers; changing course has a relatively slow but steady effect on dynamic density. In contrast, sharp drops in density indicate vertical maneuvers. Density falls abruptly at the instant when vertical separation exceeds 1000 feet. Figure 8 shows that all four pilots made vertical maneuvers. Pilots 1 and 4 (shown by squares and hexagons) changed altitude substantially earlier than Pilots 2 and 3.

With this background, it is possible to infer from Figure 8 the sequence of decisions that the pilots made. When the scenario began, density increased as all four pilots took time to detect and assess the traffic situation. Approximately 40 s into the scenario, all took steps that resolve the programmed conflict. Pilots 1 and 4 changed altitude, and Pilots 2 and 3 changed course heading. These maneuvers reduced dynamic density temporarily. For Pilot 4, density never rose above 0.1 again; this pilot had found an altitude with little traffic.

The other three pilots executed maneuvers that created secondary conflicts. Density began to increase 1–2 min after their initial maneuvers. The impending loss of separation required additional evasive maneuvers. In the end, only Pilot 2 failed to resolve the secondary conflict before it became an official accident. The virtual square-wave produced by Pilot 1—a sharp increase in density for a short period followed by a abrupt decrease—indicates a secondary conflict produced by passing through (into and out of) an altitude with nearby traffic. In contrast, Pilots 2 and 3 ran into a secondary conflict by turning into traffic.

Consequently, the time series of dynamic density provides the capability to discriminate among pilots who succeed and fail to maintain self-separation and to identify the type, time, and relative effectiveness of their individual maneuvers. When plotted together, density patterns for different pilots provide the basis for comparing efficiency as well as strategy. For example, Pilot 4 took longer to make an initial maneuver but was the only pilot to avoid...
a secondary conflict. Was this pilot's success the result of a conscious waiting strategy? Was his success related to making a vertical maneuver at an optimum time? Why was the lateral maneuver made by Pilot 2 less successful than that of Pilot 3? The pattern of density values does not answer all of these questions. Rather, it serves an equally crucial function: It inspires formulation of such questions.

Recreation, Simulation, Training, and Evaluation

Researchers in the FAA are currently developing systems that use recorded radar and computer data from en-route traffic control centers to re-create historic air traffic situations. Particular attention has been paid to situations involving a violation of separation minima and the potential task and performance characteristics associated with these error events (Rodgers & Manning, 1995; Rodgers, Manning, & Kerr, 1994). This approach requires accurate representation of airspace dynamics and aircraft proximity. The Index of Dynamic Density is a measure that is compatible with this approach. It provides dynamic information about the separation of pairs of aircraft in the airspace. Once scenarios are re-created in simulation, baseline values for dynamic density can be obtained. Alternative maneuvers and trajectories could then be evaluated by determining how factors such as the timing of events and type of maneuvers affect the pattern of index values.

Application of the Index of Dynamic Density for re-creation highlights its role in retrospective analysis. The index can also serve a proactive role in the training and education of pilots and traffic management providers. For instance, it can be used a priori to rate the difficulty of training scenarios in a more meaningful way than can simplistic descriptors such as the number of aircraft. The index could be used to give pilots and traffic management providers the opportunity to determine first-hand how complexity factors, alone and in interaction, contribute to difficulty and workload.

In a training and evaluation environment, the index may be used to rank-order the performance of different operators in a simulated airspace. In Table 1, three alternative evaluation criteria are displayed and applied to the performance of the pilots in the scenario of Figure 8. We offer these criteria and comparisons to illustrate the variety of evaluations possible with our index of dynamic density. If speed of resolution were the criterion, Pilot 1 would be deemed to have performed better than the other three. However, if efficiency were the criterion, Pilot 4 made the better strategic choice.

**SUMMARY**

We have developed an analytical approach to the assessment of dynamic density and have shown how to use this method to assess pilot performance in self-separation traffic scenarios. The index captures the factors enumerated by Pilot 1: first pilot to resolve programmed conflict
Pilot 3: second pilot to resolve programmed conflict
Pilot 2: third pilot to resolve programmed conflict
Pilot 4: last pilot to execute initial maneuver

**Table 1: Example Criteria that can be Used to Evaluate Pilot Performance from Inspection of Time Plots of Dynamic Density.**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Rank Order in Figure 6</th>
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<tbody>
<tr>
<td>Speed of resolution of programmed loss of separation</td>
<td>Pilot 1: first pilot to resolve programmed conflict</td>
</tr>
<tr>
<td>Efficiency of resolution alternatives</td>
<td>Pilot 2: last pilot to execute initial maneuver</td>
</tr>
<tr>
<td>Percentage time in low (&lt;0.5) level of dynamic density</td>
<td>Pilot 1, Pilot 4, Pilot 3, Pilot 2</td>
</tr>
</tbody>
</table>

the RTCA task force in its definition of dynamic density while remaining easy to calculate and interpret. More generally, it can be applied to any data involving a time stamp and position information for two or more moving objects. The numerical value of the index at a given time is a relative measure of dynamic density at that time. Peak values of the index contain information about the time, separation, and relative level of risk posed by nearby objects. The index can be readily extended to provide more global measures of dynamic density.

We recognize that $D$ is not the only form that an index of dynamic density might take. We offer it as one formulation that is relevant to several areas of transportation and to other human factors realms, including occupancy of confined areas such as spacecraft, or crowd evaluation at sports events or shopping malls. Given the many realms in which such an index can be useful, it is our contention that $D$ can be a tool in the armory of the practitioner and researcher alike.

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