

## A Note on the Speed-Amplitude Function in Movement Control

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**ABSTRACT.** An experiment is reported that documents the maximum average speed-amplitude relationship across the full range of motion for elbow flexion. Minimum movement time increased as a negative exponential within the movement range up to 94-97% of the maximum range of motion. At this point a discontinuity occurred with movement time increasing at an increasing rate probably due to anatomical and morphological constraints. These results suggest that the maximum average velocity-amplitude boundary to the movement speed-accuracy relationship is curvilinear. Kinematic analysis of the movements as a function of range of motion suggests that a simple pulse-step model of movement control cannot account for the present findings.

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THE RELATIONSHIP BETWEEN the speed of a movement and its resultant accuracy has enjoyed a prolonged and continuing interest among those concerned with motor control (e.g., Beggs & Howarth, 1970; Fitts, 1954; Woodworth, 1899). Indeed, this problem forms the focus of current arguments about the relationship between kinematics, kinetics, movement error, and modes of control (e.g., Hancock & Newell, in press; Meyer, Smith, & Wright, 1982; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979). These accounts have attempted to examine processes subsuming control from coherent descriptions of the interrelationship between movement amplitude, temporal duration, and some measure of the outcome in terms of an error score.

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The relationships between kinematic parameters and movement error are constrained to occur within certain boundary conditions. These are formed by the morphological and anatomical constraints of the limb(s) utilized for action and the power of resolution of the device(s) used in measurement. The few studies that have reported on a maximum average velocity-amplitude function have examined only a limited range of the possible spectrum of movement conditions. In the present note we report an experiment that documents the maximum average speed-amplitude boundary across the full range of motion for elbow flexion.

The initial investigation of the speed-amplitude limit was undertaken before the turn of the century. In their monograph, Fullerton and Cattell (1892) reported data which suggested that the minimum time to move through amplitudes from 10–70cm increased, but at a rate less than either a simple linear or a square root function. In a more recent investigation, Wadman and his colleagues reported that in the range 6–32cm the relationship was essentially of a linear form for a unidimensional arm movement (see Wadman, Dernier van der Gon, Geuze, & Mol, 1979, Figure 3). The slope of the reported linear relationship indicated a positive intercept with the ordinate. Taking both the data sets available, the projected function for maximum average velocity would commence at the origin and increase as a negative exponential as amplitude approached the limit of the limb(s) utilized for movement.

The following experiment provides an empirical examination of the maximum average velocity-amplitude function. The results suggest sources of organismic constraint to the speed-amplitude relationship. Furthermore, analysis of the kinematic parameters indicates a different mode of control with respect to this function, than has been suggested by Freund and Budingen (1978), Ghez and Vicario (1978), and Ghez (1979).

## Method

### *Subjects*

The subjects were 6 student volunteers from the University of Illinois. There were 3 females and 3 males in the group. All subjects were right-handed.

### *Apparatus*

The apparatus has been described in detail previously (Newell, Carlton, & Carlton, 1982). Briefly, the apparatus was a free moving horizontal angular displacement bar which afforded elbow flexion-extension activity through the full anatomical range of motion. The subject was seated such that the center of the elbow joint of the right arm was directly over the axis of rotation of the bar. Acceleration and displacement data were collected at a sampling rate of 1000 Hz by a PDP 11/23 computer.

### Procedure

Initially, the range of flexion motion was determined for each subject and the absolute ranges of motion calculated to represent 1, 4, 7, 20, 40, 60, 80, 94, 97 and 100% of motion. The starting position of each movement had the right forearm completely extended at the elbow joint with the upper arm-forearm complex supported in the horizontal plane perpendicular to the sagittal plane of the body. Each movement trial was initiated by a warning light followed by a start light with a variable foreperiod of 0.75, 1.50, 2.25, and 3.00 s. The subject was instructed to minimize movement time *through* each designated range of motion. A marker pole clearly indicated the amplitude target for each condition. No constraint was placed upon where the subject brought the limb to rest beyond the amplitude target, although the anatomy of the elbow joint provides natural constraints. Trials within conditions were completed approximately every 10 s.

### Experimental Design

Testing took place over 3 days. On day 1 each subject practiced 5 of the 10 conditions. The practice conditions were randomized for each subject with the constraint that there were 2 or 3 conditions in the lowest five % values of the range of motion conditions.

On day 2 subjects were assigned to complete either the 5 conditions of 1, 7, 40, 80 and 97% range of motion or the conditions of 4, 20, 60, 94 and 100%. On day 3 each subject completed the remaining percentage of range of motion conditions. The set of conditions per day was counterbalanced across subjects and within a day conditions were randomized for each subject.

There were 20 trials in each practice and experimental condition. A data reduction program derived a large set of kinematic parameters for each trial.

### Results

Figure 1a and b depicts movement time and average velocity as a function of the percentage of range of motion. The results are clear in showing that minimum movement time increases at a decreasing rate for equal increments in the range of motion. However, a discontinuity is evident between 94–97% of the range of motion. At this extreme end of the range of motion, minimum movement time is slower than would be predicted by the general function for the maximum average velocity-amplitude relationship.

A polynomial regression analysis up to the fourth degree was conducted to determine the best fitting polynomial for the movement time function. The analysis revealed that the linear, quadratic, and cubic components were all significant,  $F(1,5) = 1208.97, 8.86, 12.41, p < .05$ , respectively. The best fitting third order equation is:

$$MT = 29.33 + 7.44 (\% \text{ range}) - .10 (\% \text{ range})^2 + .00058 (\% \text{ range})^3$$

where MT is movement time (ms) and percentage range equals the percentage range of motion. When the data points from the 97% and 100% range of motion conditions were eliminated from the polynomial regression only the linear and quadratic components were significant,  $F(1,3) = 1257.62$ ,  $28.72$   $p < .05$ , respectively. The best fitting second order equation is:

$$MT = 44.26 + 4.70 (\% \text{ range}) - .019 (\% \text{ range})^2$$

The coefficient of variation for movement time (Figure 1c) indicates that variability is proportionally greater over very limited ranges of motion where the minimum movement time and maximum average velocity are small. This finding is consistent with our previous work which has revealed that the initial phases of a movement are relatively more variable than later segments of discrete arm movements (Newell et al., 1982).

An understanding of how the movements were produced in minimizing movement time through the range of motion may be gleaned from a kinematic analysis of the movement trajectories. As the range of motion increased time to peak jerk and time to peak acceleration also increased (see Figure 2a and c). Peak jerk is the peak rate of change of acceleration. Furthermore, at the shorter range of motion conditions, time to peak acceleration occurred after the subject had completed the criterion range of motion. Duration of the acceleration phase generally increased with increments in the range of motion although it is less systematic at the shorter percentage of motion conditions. This variation at the shorter range conditions is probably due to the fact that the subjects were free to vary the duration of the acceleration phase owing to the task criterion being achieved before the initial acceleration phase had been completed.

Both peak jerk and peak acceleration followed an inverted u-shaped function over the range of motion (see Figure 2b and d). Contrast of the directional shifts over the range of motion in peak acceleration and time to peak acceleration suggests some degree of trade-off between these parameters. The function for peak velocity over range of motion exhibited similar trends to that depicted for peak jerk (Figure 2b).

### Discussion

The findings demonstrate that minimum movement time increases at a decreasing rate with equal increments in the range of motion traversed. Thus the maximum speed-amplitude relationship is a negative exponential as suggested by the limited data of Fullerton and Cattell (1892). The linear speed-amplitude prediction of Wadman et al. (1979) arises presumably as a consequence of describing kinematic relationships of movement conditions confined to the middle range of motion. The speed-amplitude function sets a boundary to the speed-accuracy relationship (Hancock & Newell, in press) and is useful in understanding the constraints imposed on movement production in various practical settings.

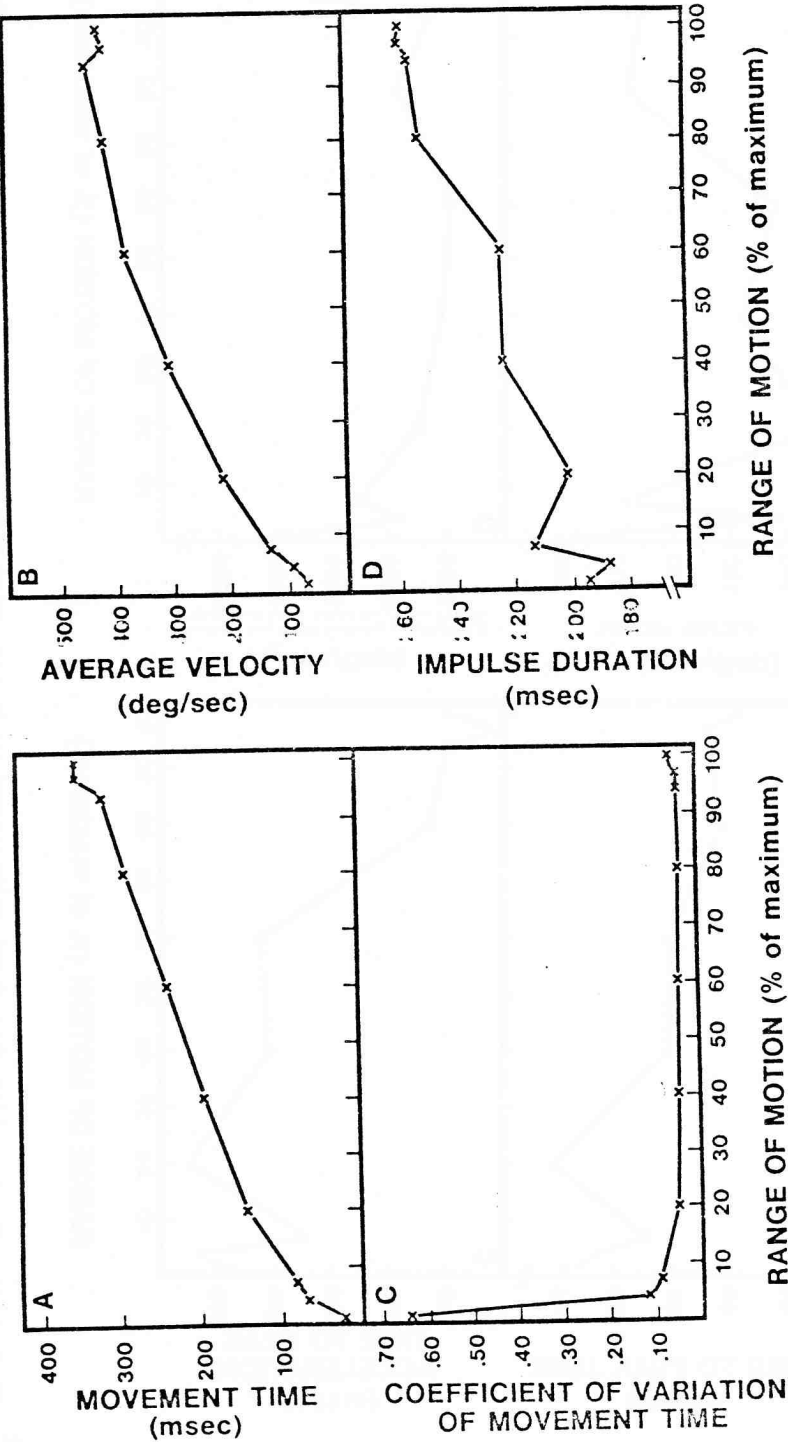


Fig. 1—(a) Mean movement time, (b) average movement velocity, (c) coefficient of variation for movement time, and (d) duration of the acceleration phase as a function of the % of range of motion.

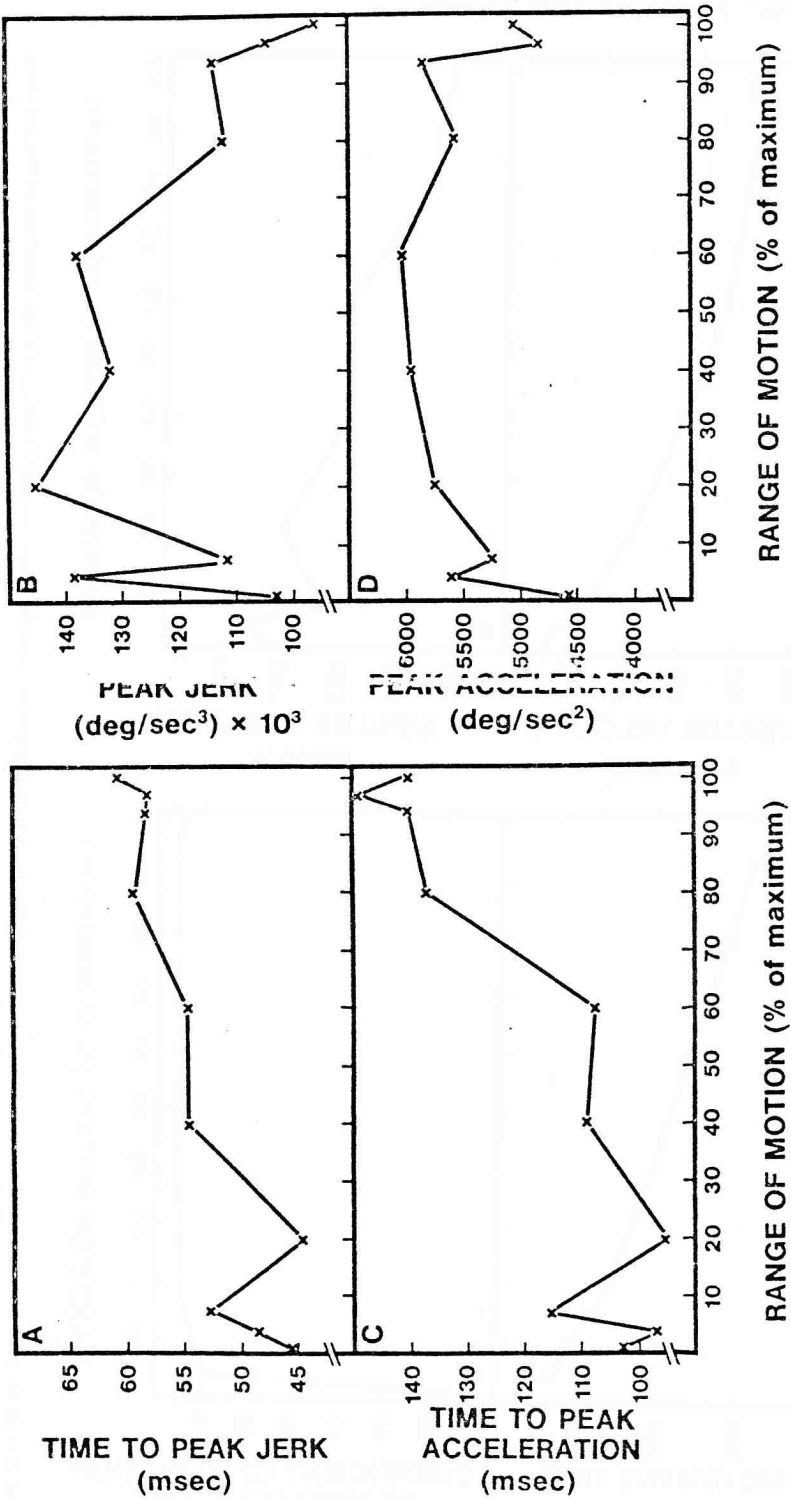


Fig. 2—(a) Mean time to peak jerk, (b) peak jerk, (c) time to peak acceleration, and (d) peak acceleration as a function of the % of range of motion.

The curvilinear speed-amplitude relationship covers the majority of the range of motion, but it breaks down as the range of motion approaches maximum. In the current experiment a discontinuity occurred between 94–97% of the range of motion. It is our estimation that the locus of such discontinuities across the range of motion will depend upon the anatomical unit(s) involved and the nature of the action undertaken.

Anatomical and morphological constraints presumably dictate the slowing of movement at the extremes of motion. It is possible that the discontinuity would occur at a slightly lower percentage of range of motion had we employed an elbow extension rather than flexion task. This is because the biceps muscle provides a natural padding to brake the movement in flexion so that the extreme of the joint motion is not achieved, whereas this natural protection is not so evident at maximum extension. However, it is probably appropriate to conjecture that the curvilinear maximum speed-amplitude function will hold to approximately 90% of the range of motion for prescribed actions. The viscoelastic forces of the elbow flexors and extensors also increase considerably toward the full range of flexion and extension, respectively (Lestienne, 1979).

The kinematic data are instructive in assessing current concepts regarding the mode of control in producing maximal effort over different ranges of motion. Freund and Budingen (1978) provided evidence that humans minimize movement time over varying ranges of motion by simply scaling up the amplitude of the force pulse while holding the duration of the impulse constant. Ghez (1979) has shown similar findings with cats in an isometric force production task and has described this mode of control in terms of a pulse-step model.

The kinematic data from the current experiment are in contrast to those of Freund and Budingen (1978) with respect to both isometric and isotonic tasks. Above some minimal amplitude and hence level of force production, our subjects systematically increased both time to peak jerk and time to peak acceleration as range of motion increased. However, close inspection of the Freund and Budingen (1978, Figure 1) data suggests that the differences may be more a matter of interpretation than the actual data produced. Our reading of the data provided by Freund and Budingen (1978, Figure 1) for an isometric task is that time to peak impulse lengthened as the peak force increased in a manner consistent with the findings of the current study. These small systematic shifts of time to peak force are also evident in the isometric studies of Ghez (1979) and Newell and Carlton (in press). The lack of quantitative analysis by Freund and Budingen may have biased their visual inspection of the isometric force-time curves.

Furthermore, in the isotonic responses examined by Freund and Budingen (1978, Figure 6) a 10% spatial error boundary was established but no report of movement error was available. Thus it is difficult to determine the degree to which a speed-accuracy trade-off occurred in the Freund and Budingen isotonic task. The task constraints imposed by

Freund and Budingen (1978) were also different from the current experiment in that the spatial error boundary was proportional to the amplitude moved in a manner similar to preserving the index of difficulty in Fitts' (1954) law. It is not surprising, therefore, that movement time was essentially invariant to increments of amplitude and that an orderly set of velocity curves emerged. The task constraints of the current experiment required the minimization of movement time in traversing a given amplitude without concern for the spatial accuracy relative to the principal direction of motion. Apparently, under these task constraints movement time is not minimized over the range of motion by simply scaling up the amplitude of the impulse.

The current experiment does not provide a basis to determine the principle for the minimization of movement time. However, our data suggest some trade-off between peak force and time to peak force and hence, average rate of acceleration as a function of the task constraints imposed. This relationship is probably consonant with principles of optimization and efficiency (see Newell, Carlton, & Hancock, 1984).

Although the findings indicate that the pulse-step model is too simplistic to accommodate the data of the current experiment, no obvious alternative candidate models emerge. It would seem, however, that models of force production need to be predicated on a broader basis than a single set of task constraints. Moreover, the data are clear in demonstrating a curvilinear maximum speed-amplitude function up to approximately 90% of the initial range of motion, and this establishes one boundary of the speed-accuracy relationship of movement control.

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