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Forgotten Moments: A Note on Skewness and Kurtosis as Influential Factors in Inferences Extrapolated from Response Distributions

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ABSTRACT. It is proposed that reliance on only the mean and standard deviation of a distribution to describe response frequency may lead to erroneous inferences concerning such distributions when skewness and kurtosis are present. After defining the first four moments of a distribution, it is demonstrated analytically that skewness and kurtosis may vary to systematically influence the mean and standard deviation of a set of related distributions. The significance of these relationships for the interpretation of differing response distributions is advanced through examples gleaned from the movement control literature. In addition, it is suggested that the use of bandwidths to select scores from a distribution for subsequent data analysis can further compound the problems of both descriptive and explanatory inference, particularly when skewness and kurtosis are features of such distribution(s). Whether or not inferential statistics are invoked, a veridical perspective of distributions is essential to meaningful data analysis.

A STANDARD EMPIRICAL strategy for researchers in motor behavior is to examine features of a dependent variable or variables through some statistical analyses of the distribution of the scores collected. The assessment is usually based on the mean of the distribution, although the standard deviation, is also calculated on many occasions to enrich the description of the sample distribution. In this paper it is proposed that reliance on only the first and second moments of a sample distribution can lead to erroneous perspectives of the distribution and as a consequence may color inferences drawn with respect to, for example, the effect of an independent variable on a dependent variable or the relationship between dependent variables. Failure to consider fully all

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elements of the sample distribution is a problem common to users of statistics in a variety of fields of study and there have been a number of recent commentaries on this situation (e.g., Tukey, 1977; Wainer & Thissen, 1981).

The mean and standard deviation are generally the only two descriptive statistics of response distributions to be reported in motor behavior research. These statistics are usually calculated from a distribution based on actual scores or a distribution of deviations from some criterion value. For example, constant error and variable error have been presumed to reflect different processes in the short term retention of simple movements (Laabs, 1973) and a measure of variability is fundamental to formulations of the movement speed-accuracy trade-off (e.g., Woodworth, 1899) and schema theory (e.g., Schmidt, 1975). To develop the case for the significance of the statistics based on third and fourth moments of the response distribution, and their scale independent derivatives namely, skewness and kurtosis respectively, an initial condition is the reiteration of the definitions of moments of a distribution. A full account of these descriptive statistics may be found in most basic statistical texts (e.g., Glass & Stanley, 1970; Hays, 1963) and as a consequence, only sufficient mathematical detail is provided to understand the fundamental components of the first four moments of a distribution together with some of their derivatives.

Moments of a Distribution of Scores

There are a variety of statistics which can be derived from a sample of scores to describe a distribution of the scores. These descriptive statistics may be separated into various measurement categories. The categories of central tendency and variability are those most frequently utilized by researchers.

The most commonly used measure of central tendency is the mean ($\overline{X}$) which is calculated by summing all the scores (X_i) from the sample and dividing by the number of scores summed (n).

$$\overline{X} = \sum_{i=1}^{n} X_i/n \tag{1}$$

The mean has a number of interesting properties. For example, when the distribution of the scores is symmetrical around the mean (e.g., Figure 1a) then the mean is equal to both the mode and the median. However, the relationships between these three measures of central tendency change when the distribution of scores is assymetrical with respect to the mean (Figure 1b). Despite the fact that skewness leads to an intricate relationship between these three measures of central tendency, it is the case that the vast majority of the studies in the motor behavior literature, report only the mean. There are alternative descriptive statistics which reflect the central tendency of a distribution, including the geometric and harmonic means, but these measures are rarely utilized.

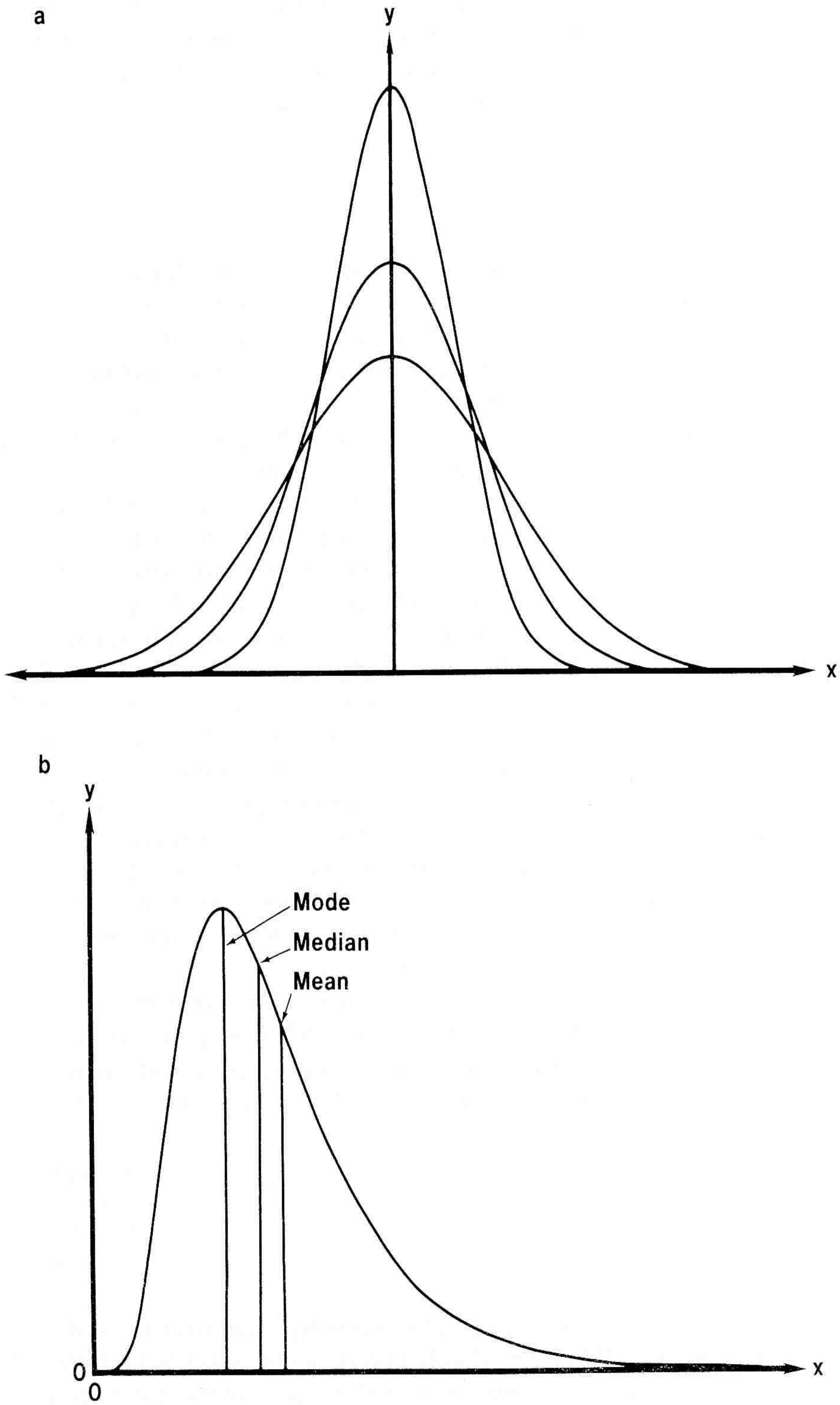

Fig. 1—(a) Response frequencies exhibiting systematic changes in kurtosis with skewness kept constant at zero. (b) A sample positively skewed distribution.

In the same way that the mean has been and is the predominate

estimate of central tendency, the standard deviation of the distribution is the principal measure of variability. The variance of the sample distribution is the sum of squared deviations of each score from the mean divided by the number of scores. The square root of the variance is the standard deviation (S_x) which is thus denoted as:

$$S_x = \left(\sum_{i=1}^{n} (X_i - \overline{X})^2/n\right)^{1/2} \tag{2}$$

The standard deviation has several properties of interest, these include the attribute that when the sample distribution is normal, as in Figure 1a, the percentage of scores which fall within any specified value of a standard deviation may be calculated. Other measures of variability include the various range scores.

The mean and standard deviation represent the statistics most commonly used to describe a distribution of scores regardless of whether inferential statistics are subsequently invoked. This approach is sufficient in cases where both the sample and population distributions are established as normal. However, it is often the case that there is some degree of asymmetry (skewness) and peakedness (kurtosis) in the population distribution curve (Glass & Stanley, 1970) and indeed under many frequency circumstances a sample may not form a normal distribution even if the parent population may do so (e.g., Fisher, 1915; Student, 1908). The measures of skewness and kurtosis are rarely reported in statistical accounts of response distributions presumably because of the implicit assumption of normality in both the population and sample distribution which makes expression of the derivatives from the third and fourth moments apparently redundant. A less generous but more realistic statement is that skewness and kurtosis are simply forgotten reflections of the third and fourth moments respectively, of the response distribution.

Skewness is the indicant of asymmetry about the mode and is formulated from the third moment (M_3) of the distribution as in essence, it reflects the average of the deviation scores raised to the third power divided by the standard deviation raised to the third power:

$$M_3 = \sum_{i=1}^{n} (X_i - \overline{X})^3/n \tag{3}$$

Leading to:

$$\text{Skewness} = \sum_{i=1}^{n} (X_i - \overline{X})^3/n/S_x^3 \tag{4}$$

Hence, if the scores are symmetrically distributed around the mean the skewness is zero. When the distribution of scores extends from the mean further toward the larger than smaller values of the distribution (Figure 1b), then the distribution is said to be positively skewed. The complement of this is negative skewness which occurs when the scores extend from the mean further towards the smaller than larger values of the distribution. Contrasts of skewness may be made across different distributions because the division of the third moment by S_x^3 in Equation (3) makes the estimate of skewness independent of the distribution scale. Skewness also leads to the mean having a different value from the mode.

The fourth moment (M_4) of the distribution is the basis for the kurtosis statistic which takes the ratio of the deviation scores raised to the fourth power to the standard deviation also raised to the fourth power.

$$M_4 = \sum_{i=1}^{n} (X_i - \overline{X})^4/n \tag{5}$$

Leading to:

$$\text{Kurtosis} = \sum_{i=1}^{n} (X_i - \overline{X})^4/n/S_x^4 \tag{6}$$

Kurtosis reflects the peakedness of the distribution, with a normal curve having a kurtosis value of 3. High peakedness or leptokurticness leads to a number greater than 3 whereas flatness or platykurticness leads to a kurtosis estimate between zero and three (see Figure 2a for examples of distributions with kurtosis). Some researchers and packaged statistical programs subtract the value 3 from the kurtosis estimate in order that zero represents the value of kurtosis when the distribution is normal.

Thus the statistics based on the third and fourth moments, namely skewness and kurtosis respectively, help define the shape of the distribution of scores. The preceding albeit brief, account should be sufficient to understand the basis of the third and fourth moments together with their mathematical derivation with respect to the mean and standard deviation of a distribution. In principle *n* moments of a distribution may be calculated but in practice moments beyond the fourth power tend to be unstable and yield little additional reliable information (Hoel, 1971).

The Relationship Between Moments of a Distribution

Although the definitions of the moments of a distribution may be found in most statistical texts, concomitant discussion of the relationship between the moments is a rarity. The major reason for this perhaps, is that the moments of distribution are not usually manipulated in some independent fashion, rather they emerge from the sample of scores collected.

It may not always be reasonable to assume that the population distributions of movement parameters are normal. That is, varying degrees of skewness and kurtosis can be inherent features of the *population* distribution. Under this situation, biases in the third and fourth moments of the sample distribution(s) may be expected to exist irrespective of biases due to sampling error. Of course, sampling error could lead to an estimate of a normal distribution when in fact the population distribution deviates from normality.

It should be noted that "The sampling variance of a moment depends on the population moment of twice the order, that is, becomes very large for higher moments, even when n is large" (Kendall & Stuart, 1977, p. 249). It is for this reason that some statisticians have questioned the practical utility of moments beyond the second power because the size of the standard error of each cumulative moment grows exponentially with the order of that moment. As a consequence, the stability of estimates of skewness and kurtosis are progressively more dependent upon the size of the sample under examination. However, researchers should be aware that this instability is inherent to a lesser degree in lower order moments whose validity depends upon a number of observations greater than that typically utilized in current motor behavior research.

Even if the population distribution is normal the sample of scores drawn from this distribution may reflect deviations from normality, particularly when the number of scores sampled is small (Editorial, 1915; Fisher, 1915; Student, 1907). An editorial in the 1915 edition of *Biometrika* provides a table of sample distribution moments as a function of sample size. It shows that for the samples utilized, estimates of skewness and kurtosis have for all practical purposes reached values indicative of normal distributions when the sample size was $n < 50$. Furthermore, with this sample size the estimates of the probable error of the standard deviation were consistent with statistical theory. However, when the sample size was less than 20, the value of the standard deviation was usually less than that of the population and skewness and kurtosis were apparent. This example again demonstrates the potential impact of sample size upon estimates of the distribution moments and also raises the issue of the degree to which any of the first four moments can be meaningfully contrasted across sample distributions without reference to the other three moments.

When more than one distribution is being analyzed, systematic variations of one or more of the moments across the distributions may be made and the impact of this manipulation on the other moments calculated. Of particular interest in this paper is the fact that systematic variations in the third and/or fourth moments across a set of distributions may influence the functions for the first and second moments of the same set of response distributions. No general rules for these effects may be formulated as they are data driven and specific to the distributions examined. However, an analytical consideration of particular hypothetical examples of the independent manipulation of skewness and kurtosis may aid in understanding of the potential significance of these forgotten moments in the analyses of data sets.

To illustrate a relationship between the above parameters we may examine properties of the Chi-squared distribution in which coefficients of skewness and kurtosis are detailed (Lancaster, 1969, p. 20). For the Chi-squared distribution of the first four moments are:

$$M_1 = n$$
$$M_2 = 2n$$
$$M_3 = 8n$$
$$M_4 = 48n + 12n^2$$

Where n is the mean. Since skewness (γ_1) is defined as

$$\gamma_1 = \mu_3/S^3 \quad \text{(from 4)}$$

Then as

$$M_2 = 2n,\ M_3 = 8n$$
$$\gamma_1 = \frac{8n}{(2n)} \tag{7}$$
$$\gamma_1 = 4/S \tag{8}$$

A similar procedure may be adopted for Kurtosis (γ_2) through reduction from the expression

$$\gamma_2 = \mu_4/\mu_2{}^{-2} - 3 = 12n^{-1} \tag{9}$$

that is,

$$\gamma_2 = 12/n = 12.2^{1/2}/(2n)^{1/2} = 12.2^{1/2}/S \tag{10}$$

Analytical examples of the relationship between distribution moments may be directly calculated from known properties of beta distributions. In beta distributions a continuous random variable X takes values in the interval (0, 1). Thus beta distributions differ from Gaussian curves in that they do not stretch to infinity. Nevertheless, beta functions are well defined and can be utilized to illustrate the relationship between moments of a distribution.

Consider the example drawn from Bury (1975, p. 336) and shown in Figure 2a and b. Beta functions have two shape parameters γ_1 and γ_2 so that the beta model can assume a variety of shapes. In Figure 2a the equal variations in shape parameters produce symmetrical functions that vary in kurtosis. The standard deviations and kurtosis estimates may be calculated from known functions for beta distributions and these are presented in Table 1. The results show that for the distributions in Figure 2a, the standard deviation decreases at a decreasing rate as the distributions vary from ($\gamma = 1$) to ($\gamma = 6$) which approaches the normal curve. This relationship is consistent with the analytical analysis presented previously which showed that the relationship between the standard deviation and kurtosis was an exponential function.

Table 1 also shows the shifts in standard deviation, skewness and kurtosis for the beta functions shown in Figure 2b. It should be noted that substantial percentage shifts in the standard deviation can arise from

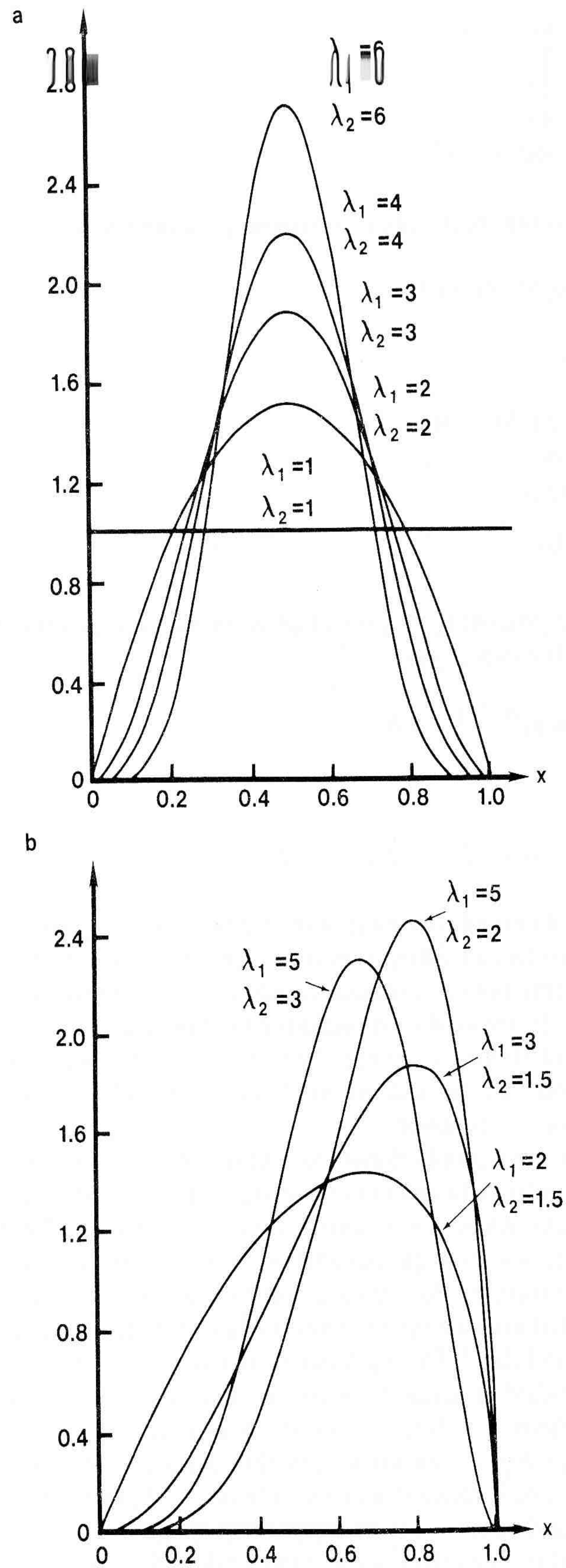

Fig. 2a and b—Sample beta distribution functions where γ_1 and γ_2 are varied as given. Abscissa represents beta distribution zero to one and ordinate represents frequency.

Table 1
The relationship between standard deviation, skewness and kurtosis for the beta function shown in Figure 2a and b.

1	2	Standard Deviation	Skewness	Kurtosis
1	1	.288	0	1.800
2	2	.224	0	2.143
3	3	.188	0	2.333
4	4	.166	0	2.455
6	6	.138	0	2.600
2	1.5	.232	− .233	2.140
3	1.5	.201	− .510	2.538
5	3	.161	− .309	2.585
5	2	.160	− .713	2.888

changes in skewness and kurtosis. Indeed, the shifts in standard deviation from this example are of a far greater order of magnitude than often reported in the motor behavior domain. Systematic changes in the third moment of a set of distributions can influence both the first and second moments of a distribution set whereas variations in the fourth moment when skewness is zero will influence only the second moment. It should be evident that changes in both third and fourth moments may combine to influence the other moments of the sample distributions.

Thus, if the third and fourth moments of the distribution could be manipulated independently while frequency was held constant, various systematic functions would emerge for the first and second moments. Obviously, the strength of the impact upon the mean and standard deviation is determined by the degree and type of skewness and kurtosis involved. The significance of these potentially independent effects of skewness and kurtosis on the first and second moments is that it demonstrates that theoretically relevant assessments of the mean and standard deviation of a distribution may only be made in light of knowledge of the bias in the third and fourth moments. For example, the same standard deviation function for a set of distributions obtained from the manipulation of an independent variable will hold different theoretical implications according to whether the distributions are normal or biased in either the third and/or fourth moments. This point is now expounded more fully in the context of the movement speed-accuracy relationship to more directly illustrate the significance of the third and fourth moments of a distribution to motor behavior research.

Example: The speed-accuracy relationship in movement control

Since the initial work of Fullerton and Cattell (1892) there have been numerous efforts to delineate the relationship between movement speed and movement accuracy. The early work of Fullerton and Cattell (1892) and Woodworth (1899) reported both constant error (*CE*) and variable error (*VE*), the first and second moments respectively, to examine the speed-accuracy function. Most subsequent investigations have either not provided constant error (e.g., Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979) or it has been combined with variable error to form a root mean square statistic (e.g., Howarth, Beggs, & Bowden, 1971) in which neither *CE* nor *VE* may be independently observed.

Parenthetically, a great many investigations have employed a target bandwidth first utilized by Woodworth (1899) and more fully exploited by Fitts (1954). One artifact of this usage is that neither *CE* nor *VE* are immediately apparent from initial measurement of response distributions. While it has been suggested that *VE* may be subsequently calculated from a percentage of target misses, *CE* shifts within the boundary constraints of the target bandwidth are not observable in the hit/miss dichotomy of response accuracy (Crossman, 1956; Welford, 1968). Moreover, the calculation of *VE* through the establishment of the total range of responses, relies on the assumption that responses are distributed normally about some mean contained within the target bandwidths.

We have demonstrated elsewhere that the assumption of normality in response distributions within a target may be based upon unfounded belief and that such distributions may be affected by biases in both third and fourth distribution moments and problems in the insufficiency of trial frequency for particular movement conditions (Hancock & Newell, in press). In a similar manner, the early work from our laboratory on the movement-speed timing-error function was based on assessment of only the first two moments (e.g., Newell, Hoshizaki, Carlton, & Halbert, 1979; Newell, Carlton, Carlton, & Halbert, 1980). To date, all attempts at decreasing the speed-accuracy function through analysis of response distributions have failed to consider the third and fourth moments, while some have based their formulations entirely on one moment, the standard deviation (Schmidt, et al., 1979). This development has occurred despite the fact that Fullerton and Cattell (1982) clearly demonstrated systematic within-subject shifts in skewness and kurtosis for response distributions of peak force-peak force variability (see Figure 3).

In the development of a space-time formulation of the speed-accuracy relationship (Hancock & Newell, in press) we have attempted to examine and accommodate the shifts in bias of the third and fourth moments into the interpretation of the mean and standard deviation of movement error as a function of movement velocity. As an example, let us consider the spatial error distributions that arise from movement time manipulations of the same amplitude (see Figure 4). The distributions range at the very low velocity condition from a high degree of peakedness (leptokurticness) with a modicum of positive skewness through a normal curve at approximately 50% of maximum average velocity to a high degree of negative skewness and a modicum of flatness (platykurt-

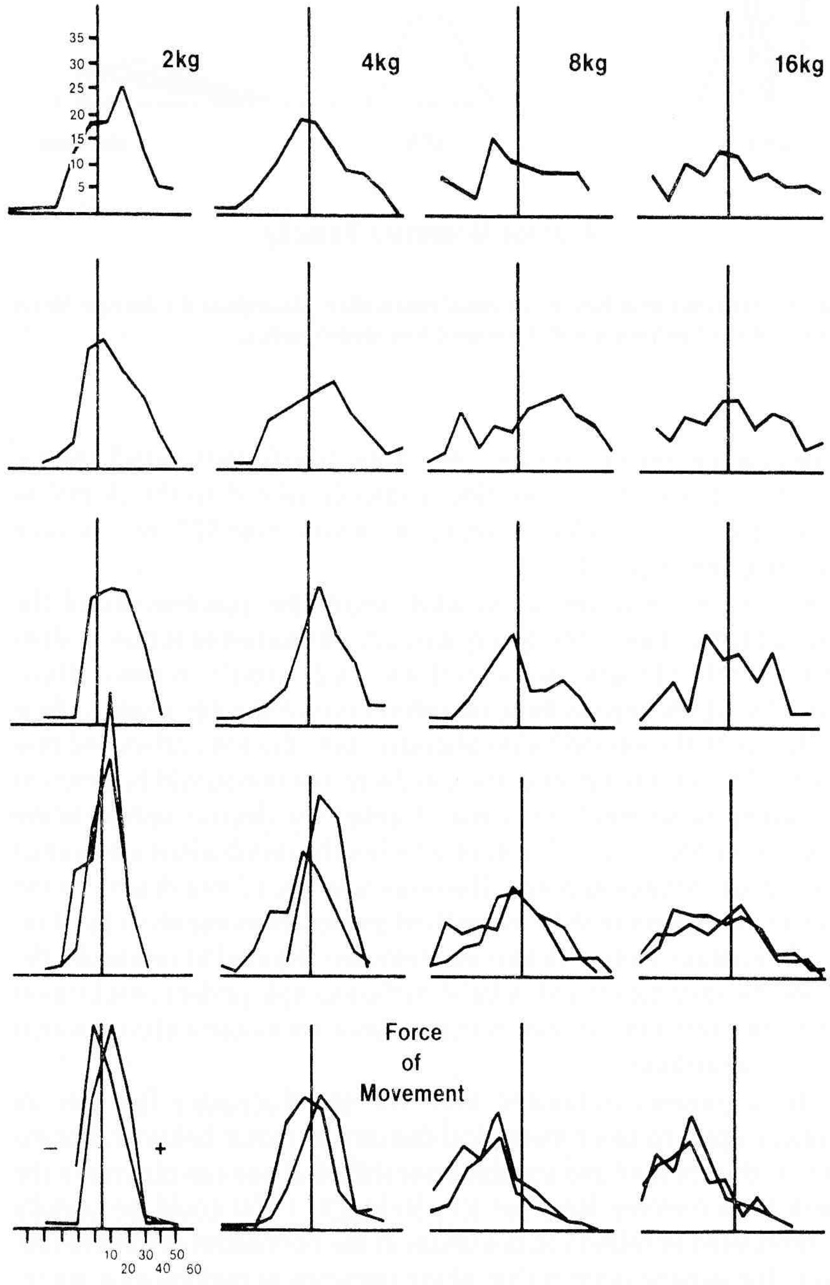

Fig. 3—Sample response distributions for five subjects demonstrating systematic variations in skewness and kurtosis as a function of criterion peak force (reproduced after Fullerton & Cattell, 1892, Figures 23–42.)

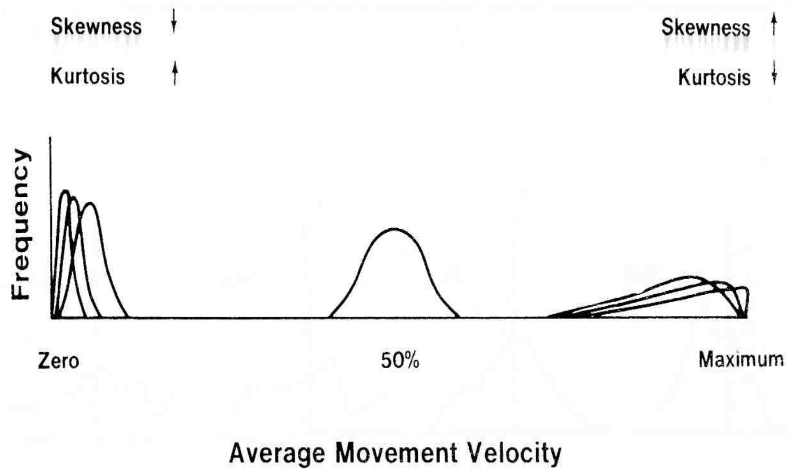

Fig. 4—Frequency distributions for equal observations throughout the Average Movement Velocity Continuum with Movement Amplitude constant.

icness) at the very high velocity conditions. The degree to which the distribution departs from normality is directly related to the degree to which the imposed velocity constraints deviate from 50% of maximum for the given amplitude.

At this time it is unclear to what degree the curvilinearity of the variability function in the speed-accuracy phenomenon is due to shifts in the third and fourth moments (Hancock & Newell, in press). However, the significance of these departures from normality reside in their relevance to the interpretation placed on both the first and second moments. The interpretation of the variability function would be different according to whether the actual distributions deviate systematically from normality (as in Figure 4) or whether the distributions are normal thoughout the velocity range. This is because one cannot determine the exact contribution of shifts in the third and fourth moments to the standard deviation. Hence all four moments are required to veridically describe the distribution and, relative to the example under consideration here, the impact of the velocity manipulation on movement error within a given amplitude.

The arguments elaborated from the speed-accuracy function inevitably apply to other theoretical domains in motor behavior. For example, the constant and variable error shifts that one can observe in the short term memory literature (cf., Stelmach, 1974) could be usefully reinterpreted in relation to deviations in the normalities of the distribution. The general point is that where extremes of performance are investigated biases in the third and fourth moments are most likely to occur, irrespective of the interval scale employed to measure performance. It is not that biases occur due to poor sampling of scores from the population (although this is possible), but rather that biases in skewness and kurtosis are likely to reflect the population distribution for many performance measures in motor behavior research. Furthermore, the problems which stem from forgetting the third and fourth moments are compounded when a criterion bandwidth is employed to select trials from the total distribution for subsequent data analysis. It is to this issue that we now turn.

Using a Criterion Bandwidth to Select Trials for Subsequent Analysis

There is a trend for researchers to utilize a bandwidth around a task criterion as a means of establishing a group of trials at a set performance level with reduced variability (e.g., Carlton & Newell, 1979; Schmidt & Sherwood, 1982). This tactic is intuitively appealing as it seems initially to ensure that performance on one independent variable is constant around a set criterion, thus enabling comparisons between non-overlapping levels of that variable on some dependent variable. In this section we show that the validity of this technique rests in part on the representativeness of the distribution of scores obtained from all trials performed.

There are several ways in which variations in the overall distribution of the independent variable might lead to problems in interpretation. Consider the distributions plotted in Figure 5. In this example, each separate group (A, B, and C) has its mean within the bandwidth and over all the trials produced, has the same standard deviation with equal kurtosis and skewness of zero. Note that there is a shift in the mean performance of the overall distribution of each group of scores. This small shift in mean performance also has an impact on the distribution of the scores within the bandwidth. In Figure 5 the variability of scores from distributions A and C within the bandwidth will be smaller than the variability of the scores from distribution B within the bandwidth, due to the biases in skewness and kurtosis that now exist. Hence, although the standard deviation of the three groups is identical for all the trials produced, the mean shift in performance will produce differences in estimates of variability of the scores within the bandwidth. This problem is compounded if the mean of the overall distribution falls outside the criterion bandwidth selected, which it could well do given the extreme range effects that often occur in motor behavior research (e.g., Fullerton & Cattell, 1892; Woodworth, 1899).

The problem raised above may become even more complex when the overall distribution of the criterion variable deviates from normality. The distribution could be skewed regardless of whether there is a mean performance difference between groups. It has been shown that skewness is most likely to occur when subjects are performing at the extreme levels of an independent variable because the majority of scores will tend to fall toward the middle range of that variable. Differences in skewness of the overall distribution of trials will have a subsequent impact upon both the mean and standard deviation of the trials that fall within the bandwidth. Thus the usefulness of selecting trials through a bandwidth criterion depends upon an understanding of the relationship between the trials in the bandwidth *and* the total distribution.

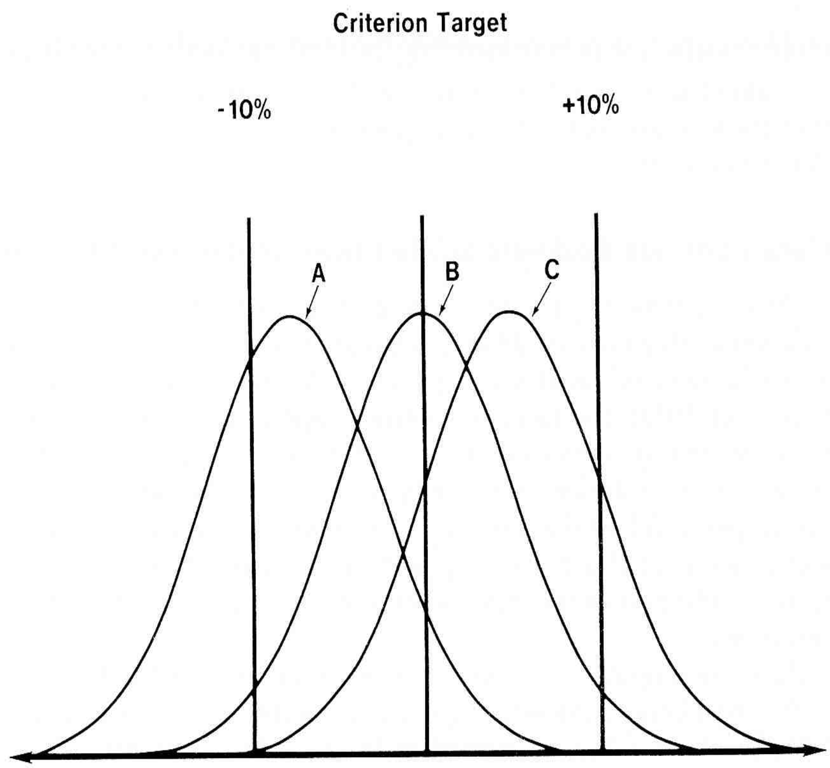

Fig. 5—Sample distributions (A, B & C) in relation to target criterion and ±10% bandwidth.

The degree to which the bandwidth trials are representative of the overall distribution is also questioned by the practice of varying the number of trials in the overall distribution. This occurs when experimenters keep the subject performing at the task until an a priori number of trials fall within the bandwidth criterion. The number of trials required to reach criterion could well vary according to condition and where in the overall distribution of data the criterion bandwidth is located. It is also worth repeating that, irrespective of the above concern, reliable estimates of the moments of a distribution can also be negated by the collection of a small number of data points for the sample set.

Obviously, the problems raised above can be minimized or magnified by the size of the bandwidth utilized to select trials for subsequent analysis. A bandwidth of ±10% of the criterion is a standard range selected although sometimes the bandwidth is wider. The fact that statistically significant mean performance difference can occur between trials selected on a 10% bandwidth basis suggests that the concerns raised in this section are not merely a hypothetical problem. Indeed, utilizing a bandwidth to select scores for subsequent analysis only highlights the problems which can stem from forgetting the impact of deviations in the third and fourth moments of a distribution.

Concluding Comments

In this communication we have illustrated a few of the problems that can emerge in statistical analysis when the third and fourth moments remain forgotten characteristics of a distribution of scores. The traditional approach of relying solely on the mean and/or the standard deviation as descriptive statistics is only appropriate if the distribution is normal and even this can be misleading when a criterion bandwidth is employed to select trials for analysis. Whether inferential statistics are invoked or not, a veridical perspective of the *distributions* at hand is essential to meaningful data analysis.

Understanding the bias in the third and fourth moments of the distribution will contribute significantly to this perspective. Graphical procedures have been developed to accommodate comprehensive management and presentation of distributions (Tukey, 1977; Wainer & Thissen, 1981) and some of these procedures are now available in computerized statistical packages. Thus there are no practical grounds to constrain the theoretical advantage of analyzing the third and fourth moments and their derivatives.

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