

## Kinetic Analysis of Response Variability

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Data pertaining to the variability of human force production as a function of the amount of force generated are synthesized in order to understand the basis for the discrepancies in previously reported estimates of this relation. It is observed that the search for a single force variability function is of limited value because a variety of functions can emerge according to the constraints imposed upon the subject during response production. Typically, however, within-subject force variability increases at a negatively accelerating rate with equal increments of force produced. Linear scaling of peak force and impulse, as proposed by the pulse-step and motor-output variability models, appears too simplistic to accommodate the manner in which subjects actually achieve reduction in response variability in force production. Rather, there is an individual specific rate of force production that minimizes variability of peak force or impulse for any given set of task constraints.

How voluntary movements are generated and controlled has been of concern to the experimental psychologist since before the turn of the century (e.g., Fullerton & Cattell, 1892; Woodworth, 1899). Subsequent studies have focused on the rapidity and accuracy with which discrete and repetitive movements may be accomplished (e.g., Fitts, 1954; Howarth, Beggs, & Bowden, 1971; Philip, 1936). At present, this orientation is recognized as a kinematic approach to movement accuracy, where kinematic parameters are derived from the dimensions of length and time and include parameters such as displacement, velocity, and acceleration. A complementary approach has been the analysis of concomitant kinetic parameters which are obtained from the units of mass, force, and time, and often include both impulse and peak force measures. Implicit in early studies (e.g., Fullerton & Cattell, 1892) and explicit in more recent work (e.g., Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979) is the recognition of a determinant connection between kinematic and kinetic de-

scriptions of response outcome, although the precise formulation of such a link has yet to emerge.

In the present article, we review the kinetic evidence pertaining to the relation between the force with which movements are generated and the variability that accrues from the repetitive engagement in such action. Our analysis is based upon isometric tasks, where ostensibly no movement occurs, and isotonic tasks, which by definition require positional changes of the performer. Although these two classes of tasks present differing control problems, the functional relation between the force generated and its subsequent variability should possess similarities across task categories. In isotonic tasks the kinetic description should also be congruent with kinematic analyses of the movement speed-accuracy relationship (e.g., Schmidt et al., 1979). In this regard the present work may be viewed as a companion to our space-time analysis of movement accuracy (Hancock & Newell, in press).

Understanding the nature of response variability has important practical and theoretical implications for motor control. For example, in many movement tasks such variability is viewed as the major limiting factor in performance, and the relative scale of variability often may be viewed as the basis to differentiate between skilled and unskilled performers. Theoretically, response variability expressed in either kinematic or kinetic terms has been viewed as a reflection of limitations in the

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neuromuscular system. These have been identified variously with time constraints of feedback loops in the control of movement (e.g., Crossman & Goodeve, 1963; Keele, 1968), noise of the neuromuscular system with respect to information transmission (e.g., Fitts, 1954), and the inherent noise of the motor apparatus (e.g., Schmidt et al., 1979). A detailed analysis of kinetic response variability could, therefore, contribute further insights into potential sources of limitation within the neuromuscular system and current models of force production.

#### Description of Kinetic Response Variability

There have been a number of attempts to describe the relation between force and force variability using a variety of motor tasks. Interest in this problem has been sporadic with each era of research activity taking place in seeming isolation with respect to previous examinations. In this section we review the studies that have provided bases for the proposed variability functions with a view to accommodating various inconsistencies that have emerged across experimental situations.

It should be recognized that the usual index of response variability is the standard deviation ( $\sigma$ ) of a given response parameter over repeated trials at a given task condition rather than the variance ( $\sigma^2$ ), as might be expected. The standard deviation may be calculated on the actual values of the designated response parameter, such as the peak force produced, or it may be calculated on the deviations of this response parameter from the respective criterion, such as the criterion peak force. The latter approach leads to a variable traditionally labeled as variable error. Of course, the standard deviation of the actual response parameter and its derived error is the same, because the subtraction of a constant (the criterion value) does not influence the variance or standard deviation of the response distribution.

The response parameter typically examined in isometric tasks is the peak force generated for a particular trial. This is defined as the highest force value attained. The criterion force levels used in experiments represent some percentage of the maximum peak force that can be generated by the muscle group tested. Thus, reference to *peak force* represents the highest

force value achieved on one or a set of responses, and *maximal peak force* represents the maximum peak force attained by the subject under the imposed task constraints. As is outlined later, the maximum peak force developed by a particular muscle group is dependent on the time available to produce that force.

We begin by considering force production in isometric tasks, which are generally considered simpler than isotonic tasks from the perspective of understanding movement control in that changes in limb displacement and the resulting change in muscle length do not occur. Force production in isotonic tasks is subsequently examined to provide a foundation for the emergent force variability function which is consistent across the various force production tasks.

#### Isometric Tasks

Perhaps the earliest and certainly one of the most comprehensive studies of response variability is rarely acknowledged in 20th century accounts of the problem. This is probably because the treatise of Fullerton and Cattell (1892) appears superficially to focus on the problems of perception in a traditional psychophysical analysis of the spatial, temporal, and force characteristics related to movement production. However, examination of the numerous experiments conducted by Fullerton and Cattell indicates that one of the primary experimental paradigms used was the method of average error. This procedure requires subjects to produce a number of responses that attempt to match a designated criterion and, in essence, is the same procedure used in contemporary examinations of the force variability function.

The experiments conducted by Fullerton and Cattell (1892) examined the validity of Weber's (cited in Herrnstein & Boring, 1965) Law in the movement domain. Weber's Law holds that:

$$\Delta s = ks, \quad (1)$$

where  $s$  refers to stimulus intensity and  $k$  is a constant. A Weberian interpretation of force production would suggest that the variability of force output ( $\sigma$ ) is proportional to the absolute level of force production (Fullerton & Cattell, 1892). This would be represented by

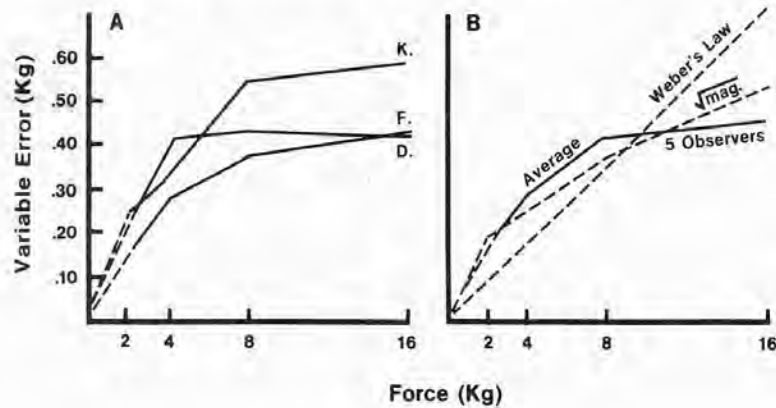


Figure 1. Relation between variable error to force magnitude for three individual subjects (a) and group average (b). (Adapted from Fullerton & Cattell, 1892, Figures 19 and 20.)

a linear relation between force variability ( $\sigma$ ) and force produced and a constant Weber ratio, defined as variability of force ( $\sigma$ ) divided by the mean force. This is more generally referred to as the coefficient of variation. The force parameter usually examined in isometric tasks is peak force although other force parameters may also be used (e.g., rate of force production, time to peak force, and impulse).

Fullerton and Cattell (1892) constructed a dynamometer with a handle attached to a heavy spring so that the subject activated predominantly the elbow flexors in response production. The task was not strictly isometric in that the handle arrangement allowed some movement, and it contained spring resistance, which presumably varied with the amount of force exerted on the handle. The distance moved at the highest force level was approximately 3.6 cm. Subjects produced a series of pulls over a range of peak force values (2 to 16 kg). The relation between the variable error and force magnitude is shown in Figure 1. Figure 1a shows the individual force variability functions for 3 of the 5 subjects measured. Figure 1b depicts the average curve for the five subjects together with the prediction associated with Weber's Law and a square root function that Fullerton and Cattell proposed apparently a posteriori to their experiments on movement production.

The square root formulation predicts that the error of observation, or the variability of repeated movements, is proportional to the square root of the stimulus. The rationale advanced by Fullerton and Cattell (1892) for such

a relation was based on probability theory. When one unit is estimated, whether it be s, cm, or kg, an error of observation will result. When two units are estimated, or produced as in force studies, each separate unit will be subject to its own error. Only if these errors are in the same direction and of the same magnitude will the observed error double, thus supporting Weber's Law. If the errors are in opposite directions, however, they would partly counterbalance each other, and the average error in estimating two units would not be twice as large as the estimation of one unit. Instead, the sum of the errors would be expressed as the average error of each unit multiplied by the square root of the stimulus magnitude. This can be expressed formally as the statistical additivity of variance with the assumption of independence:

$$\sigma_{nx}^2 = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \dots + \sigma_{x_n}^2, \quad (2)$$

hence,

$$\sigma_{nx}^2 = N\sigma_x^2 \quad (3)$$

and

$$\sigma_{nx} = \sigma_x \sqrt{N}, \quad (4)$$

where 1, 2, 3, . . . ,  $N$  refers to the magnitude of the unit estimated.

Figure 1 illustrates that the force variability function follows more closely the square root function than the linear prediction of Weber's Law. The relation between force and force variability is also reflected by a curvilinear function relating force and the coefficient of variation (obtained from values presented in Fullerton & Cattell, 1892, Table 11). The curvilinear coefficient of variation for force pro-



duction is consistent with functions related to a variety of sensory modalities including tone intensity (Riesz, 1928), lifted weight (Engen, 1971), and others (see Corso, 1967).

The average constant error function for peak force is depicted in Figure 2. At low force levels, subjects tend to overshoot and produce a force that is greater than the criterion. As the criterion force increases, the value of the constant error decreases through zero to an increasing level of negative constant error. The constant error and variable error of force production reflect the first and second moments of the error distribution. Fullerton and Cattell (1892, Figures 23 through 42) also provided representative frequency distributions for individual subjects at each force level. These vary, apparently, from high leptokurticness at low forces to a modicum of platykurticness at high force levels. This suggests that the error distributions depart from normality and require additional information beyond the second moment to adequately describe the response distribution (Newell & Hancock, in press). In summary, the analysis by Fullerton and Cattell of the relation between force variability and force level, together with similar examinations of the spatial and temporal elements of response variability, provided an initial refutation of the applicability of Weber's Law to the movement domain.

Analysis of the relation between force and force variability, however, was not taken up again directly until the practical demands of World War II stimulated a need for the understanding of human-machine interaction (cf., Fitts, 1947, 1951; Hick & Bates, 1950). The study of factors influencing the control and efficiency of machine levers and knobs helped promote an interest in force production during the execution of movement while subjects were engaged in military and industrial motor tasks. The primary focus of this research was to understand the relation of control dynamics to the accuracy with which movements could be generated with special reference to the control of aircraft. Within this milieu, Hick (1945) examined the precision of producing increments and decrements of force in an isometric task. A lever control with minimal movement tolerance (1 mm) was used to produce tracking responses to targets that were subject to step inputs.

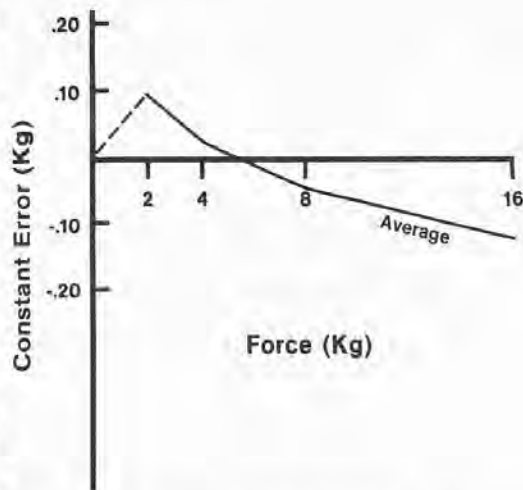


Figure 2. Relationship between constant error and force magnitude. (Adapted from Fullerton & Cattell, 1892, Figure 22.)

Hick (1945) obtained a constant error function similar to that reported by Fullerton and Cattell (1892) and showed constant error to be magnified in a task requiring relaxations to reduce force from an initial preload level. Overshooting was also greater with the imposition of an initial preload on to the typical zero load baseline. There was negligible variation in the coefficient of variation over the range of forces studied (0 to 2.27 kg). In addition, the coefficient of variation was apparently unaffected by the baseline manipulation when viewed as a percentage of the stimulus response. However, the absolute amount of variability increased with force level. This held across different baseline forces and also with step forces of 0.5 and 1 lb (0.23 to 0.45 kg). The variation in responses produced did not appear to follow a curvilinear function. This might be accounted for by the relatively few force conditions manipulated and the nontraditional preload condition used. The range of forces manipulated was small but similar to the range for which Fullerton and Cattell (1892) had previously shown effects. In summary, Hick's (1945) experiment demonstrated some interesting operational manipulations with which to approach the force variability issue. From a theoretical standpoint, however, the study raised more questions than it answered.

Jenkins (1947a) pursued the accuracy of force production in various types of aircraft

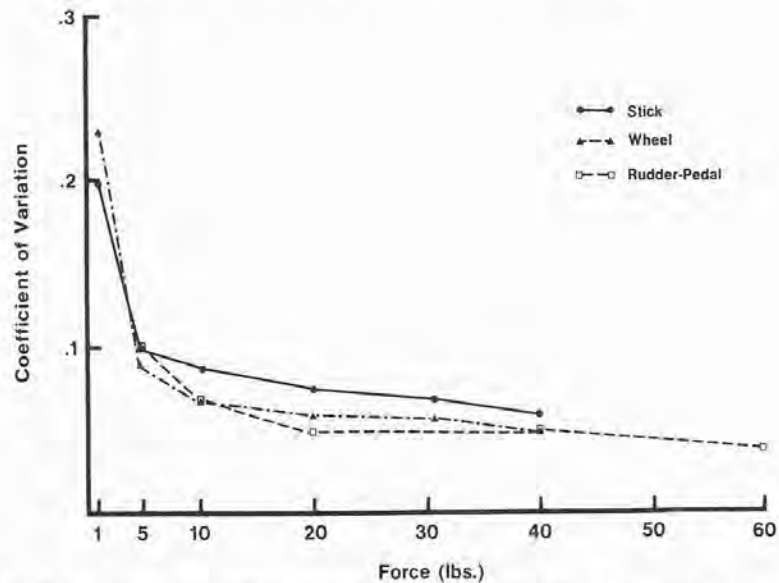


Figure 3. Coefficient of variation of force production as a function of force magnitude. (From "The Discrimination and Reproduction of Motor Adjustments with Various Types of Aircraft Controls" by W. O. Jenkins, 1947, *American Journal of Psychology*, 60, pp. 397-406. Copyright 1947 by the University of Illinois Press. Adapted by permission.)

controls, namely stick, wheel, and rudder/pedal. The forces generated ranged from 1 to 60 lb (0.45 to 27.22 kg), and the apparatus allowed only 0.5 in (1.27 cm) of movement under the most extreme force. The force range used was dependent on the task, and a smaller force was produced with the stick control compared with the rudder and wheel controls. The coefficient of variation for the three tasks, that is standard deviation of force divided by the mean force, is depicted in Figure 3. The curvilinear functions are similar to those obtained by Fullerton and Cattell (1892).

Figure 4 represents the coefficient of variation obtained for the four different actions produced with the stick control. Force production was examined for forward, backward, and left or right stick movements. The same curvilinear function was found for each of these movements, with force variability increasing even at the highest force levels. The constant error functions also showed the shift from positive to negative values with increments of force. Jenkins (1947a) noted that the Weber fraction or ratio decreased markedly as force production increased from 1 to 10 lb (0.45 to 4.54 kg). However, beyond this value Weber's Law held moderately well, although strict proportionality never was actually attained. The systematic change in the ratio im-

plies a progressive change in slope such that the overall function has the property of an initial steep increase followed by progressive flattening of the slope. Later in this article, we refer back to such an overall function as suggested by Jenkins' data.

The force variability function obtained by Jenkins appears to be of an exponential morphology, with the change in variability being greatest at low force values. This is demonstrated in both the coefficient of variation and standard deviation functions. The nonproportional relation between force level and force variability was subsequently found by Noble and Bahrick (1956) and Provins (1957), using comparable isometric force generation tasks. These findings are especially evident when the data are reanalysed and the variability of responses are divided by the force produced, yielding the Weber ratio or coefficient of variation. Collectively, the data of the immediate post-war studies indicate that force variability increases at a decreasing rate with equal increments of force, the general function originally demonstrated by Fullerton and Cattell (1892).

Following the 1950s, there seems to have been a dearth of empirical research concerning the force variability relation. However, as we review in the section on isotonic tasks, Bahrick

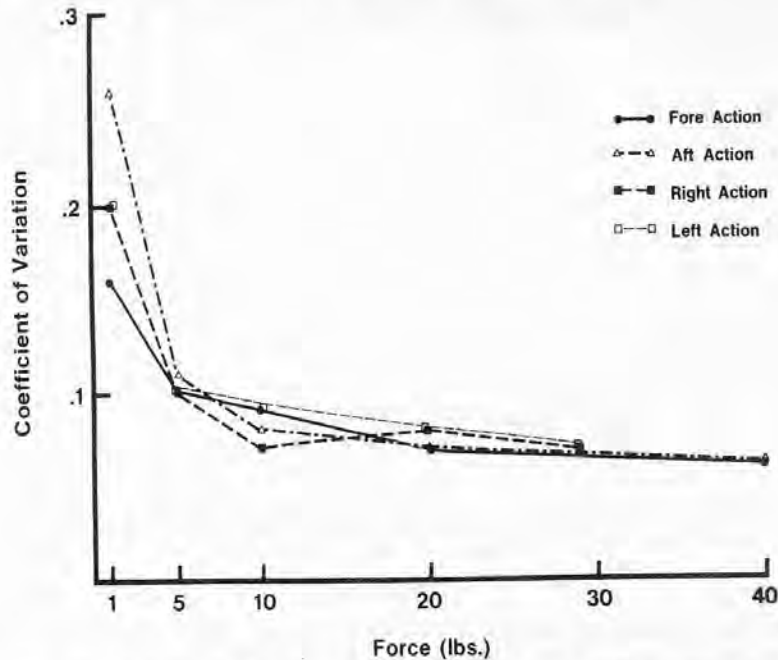


Figure 4. Coefficient of variation as a function of movement direction and force magnitude. (From "The Discrimination and Reproduction of Motor Adjustments with Various Types of Aircraft Controls" by W. O. Jenkins, 1947, *American Journal of Psychology*, 60, pp. 397-406. Copyright 1947 by the University of Illinois Press. Adapted by permission.)

and colleagues attempted to go beyond mere description and to reach an understanding of the relative contribution of response parameters to response variability (e.g., Bahrck, Bennett, & Fitts, 1955; Bahrck, Fitts, & Schneider, 1955).

Schmidt et al. (1979) recently revitalized interest in the description of the force variability relation through their model of motor-output variability. The central prediction of this model is that within-subject variability of force is proportional to the force produced. This tenet simply reiterates Weber's Law, a formulation that is inconsistent with the work reviewed previously on the relation between force variability and force level. Although the experimental methodology that Schmidt and his colleagues used is essentially identical to that used in the previously reviewed studies, there is one difference—the motor-output variability model emphasizes rapid force generation. Most of the earlier studies did not specify rate of force production or time to peak force. It is possible that the psychophysical orientation led to slower rates of force production than those emanating from Schmidt's experiments. However, it should be noted that although Schmidt and colleagues claim to have used quick contractions, one

cannot verify the meaning of quick because time to peak force was never reported.

In support of the motor-output variability model, Schmidt et al. (1979) presented several sets of data indicating a linear relation between variability and absolute level of various movement parameters, including direct evidence from isometric tasks. Schmidt et al. (1979) reported two experiments in which subjects exerted isometric forces to shoot a dot on an oscilloscope screen to a height proportional to the criterion peak force. The range of forces examined in the first experiment was 0.19 to 1.13 kg; in the second experiment, the range was 2.2 to 13.9 kg. The results from both experiments showed a strong linear relation between the within-subject variability in force and the amount of force produced. The within-subject correlations for these two variables were high and ranged from .91 to .99 and .84 to .98, respectively, for the two experiments. The slopes and intercepts for the two regression lines were essentially identical. These data are inconsistent with a curvilinear relation that the earlier studies might have led us to anticipate, given that the force range for the two experiments cover the strong curvilinear component of the force variability function as reported by, for example, Jenkins (1947a).

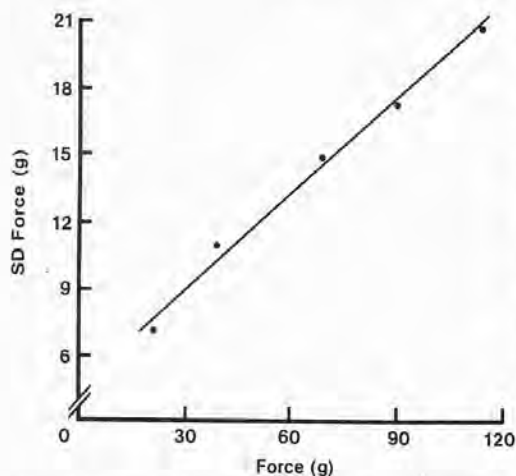


Figure 5. Force variability as a function of force magnitude. (The scaling upon the ordinate is different from the original report as that inadvertently failed to include the break in the ordinate scale. Sherwood, personal communication. From "The Relationship Between Force and Force Variability in Minimal and Near-Maximal States and Dynamic Contractions" by D. E. Sherwood and R. A. Schmidt, 1980, *Journal of Motor Behavior*, 12, 75-89. Copyright 1980 by the Helen Dwight Reid Educational Foundation. Reprinted by permission.)

Sherwood and Schmidt (1980), in an attempt to examine the force variability function beyond mid-range values, used force values at both the lower and upper ranges of force production. At lower force levels (0.02-0.116 kg), a linear relation was again found between force and force variability, with within-subject correlations of .99, .99, .97, and .97, respectively, for the four subjects tested (Figure 5). The intercept appeared to be on the order of 0.004 kg, which is somewhat smaller than that found in earlier experiments (Schmidt et al., 1979). However, there appears to be some discrepancy between the illustrated value of the intercept and that which was reported in the text of the article (i.e., 0.011 kg). Interestingly, although the function appeared to remain linear at these low force levels, the slope of the function was more than double the value obtained in their previous studies (Schmidt et al., 1979). These relative changes of slope accord with Jenkins, (1947a, 1947b) data described previously. It would appear from comparison of these data sets that the assumption of linearity up to mid-range force values is somewhat tenuous. This is particularly true because the intercept has been shown to be essentially zero (0.004 kg) which, when coupled with linearity, leads to equal Weber ratios or a constant coefficient of variation across force levels.

As our synthesis has indicated, the majority of the data sets are inconsistent with the finding of a linear and proportional relation between force and force variability, including data described from experiments by Sherwood and Schmidt (1980). Schmidt and Sherwood (1982) subsequently modified this prediction derived from the motor-output variability model. They did so not on the basis of a re-analysis of the extant force variability experiments reviewed here but in view of an additional experiment that showed that force variability increased linearly up to approximately 65% of maximum; however, at force levels higher than this point, variability decreased. In essence, the motor-output variability predictions were modified to encapsulate an inverted-U-shaped function, a relation that Schmidt and Sherwood (1982) also showed for movement accuracy. The major feature of this proposal is that variability decreases at levels of force production higher than 65% of maximum as shown in Figure 6.

It is difficult to contrast directly the earlier force studies with the experiments of Sherwood and Schmidt (1980), because previous work reported in this section did not provide an estimation of maximum force production for the respective tasks. Consequently, it cannot be ascertained where each estimate of force variability should be located on the relative force continuum for a given muscle group. However, there are some features of the earlier

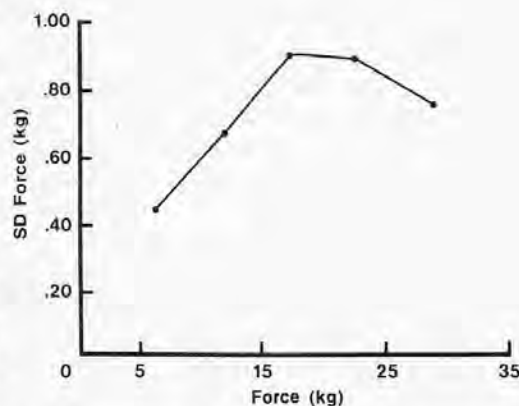


Figure 6. Force variability as a function of force magnitude. (From "The Relationship Between Force and Force Variability in Minimal and Near-Maximal States and Dynamic Contractions" by D. E. Sherwood and R. A. Schmidt, 1980, *Journal of Motor Behavior*, 12, 75-89. Copyright 1980 by the Helen Dwight Reid Educational Foundation. Reprinted by permission.)



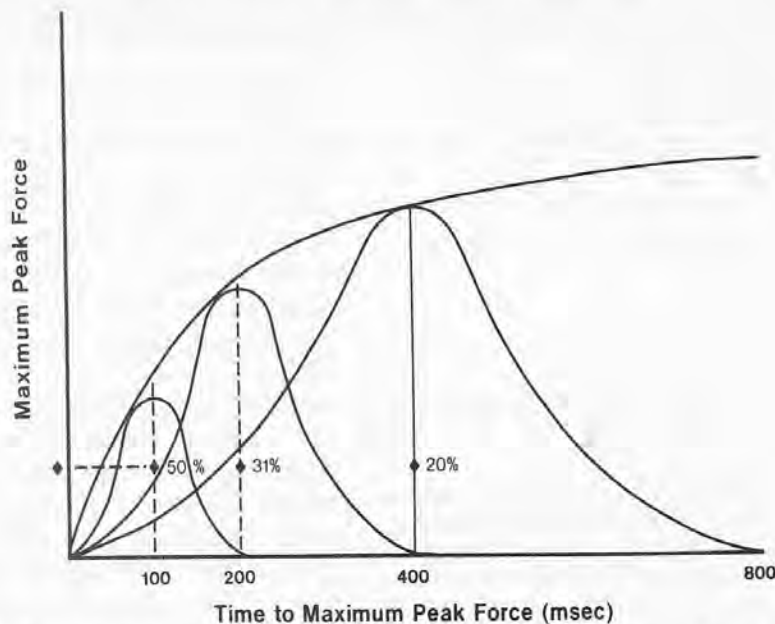


Figure 7. Projected relation between maximum peak force and time to maximum peak force. (Three hypothetical impulses are shown with times to maximum peak force of 100, 200, and 400 ms to illustrate how the maximum peak force function emerges. From "On the Relationship Between Force and Force Variability in Isometric Tasks" by K. M. Newell and L. G. Carlton, in press, *Journal of Motor Behavior*. Copyright by the Helen Dwight Reid Educational Foundation. Reprinted by permission.)

force studies, particularly those of Jenkins (1947a) and Noble and Bahrck (1956), that suggest they are within the range of percentage muscular output used by Schmidt and Sherwood. First, Jenkins (1947a) used absolute force values similar to those of Sherwood and Schmidt (1980). The arm action does not appear to differ substantially between the two studies. Second, the earlier investigations reported in this section demonstrate a relatively flat function for force variability beyond some given force values (e.g., beyond 10 lb in Jenkins, 1947a). Consequently, the range of forces examined does not appear to account for the respective differences illustrated. Rather, the difference appears to center on the matter of veridical description.

Thus, the variability of peak force as a function of peak force has been claimed to be an increasing square root function (Fullerton & Cattell, 1892), a nonproportional but increasing function (e.g., Jenkins, 1947a), a linear function (Schmidt et al., 1979), an inverted-U-shaped function (Sherwood & Schmidt, 1980), and a J-shaped function (Shea, Northam, Beach, & Howard, 1983). There are a number of experimental factors that could influence estimates of peak force variability as a function of peak force (e.g., transfer effects; Poulton, 1973), insufficient force levels to ad-

equately describe a function, and insufficient data points at any force level to obtain a veridical estimate of variability (Fisher, 1915). Surprisingly, all the previous work reviewed has failed to consider force variables other than peak force despite the concern for it (e.g., Schmidt et al., 1979). The failure to control or accommodate other force variables may be responsible for the discrepant estimates of the force variability function.

Estimates of the force variability function are implicitly based upon a force continuum from 0% to 100% of maximum force for the given isometric action. There is, however, no single maximum peak force, in that the time allowed for peak force to be generated determines the maximum peak force that can be obtained. Data from a recent experiment (Newell & Carlton, in press) suggests that maximum peak force increases at a negatively accelerating rate with increments in time to peak force as shown in Figure 7. Higher maximum peak force estimates can be obtained with a longer time available to achieve peak force. In fact, data from Kamen (1983) suggest that a contraction duration of the order of 2 s is required before the maximal peak isometric force is reached (see also Clark, 1968), although these extended contractions may induce the artifact of subjects generating torques



from body segments other than that isolated for the experimental task.

The significance of the maximum peak force function is that it suggests that estimates of peak force variability as a function of peak force need to be based upon a constant, or at least known, time to peak force. For example, if Figure 7 defines the relation between maximum force and time to peak force, a shift in time to peak force from 100 to 200 ms changes the peak force value of  $X$  from 50% to 30% of maximum. The impact is that peak force variability will be reduced by the subject lengthening the time to peak force, as the criterion force now represents a smaller percentage of maximum as determined by the modified time to peak force.

The impact of modulation of time to peak force on the estimate of the force variability function was tested by Newell and Carlton (in press). Subjects produced a range of peak forces (2½% to 90% of maximum), with a fixed time to peak force for an isometric action requiring activation of the elbow flexors. The results (see Figure 8a) showed that peak force variability increased as an exponential function of percentage of maximum peak force for the criterion time to peak force. However, as Figure 8b reveals, subjects systematically shifted on average the time to peak force as a function of the force level being produced. Only in the middle of the force range were subjects consistent in time to peak force. At the very low 2½% of maximum force level, subjects lengthened time to peak force beyond the criteria 200 ms. At the upper end of the force continuum, time to peak force was also increased, thus in effect reducing the percentage of maximum peak force that the criterion value represented and, as a consequence, peak force variability.

One method of ascertaining the impact of shifts in time to peak force upon peak force variability is to minimize the contribution of changes in time to peak force across trials. This was accomplished by multiplying the peak force attained on each trial by the attained time to peak force for the respective trial, divided by the imposed time to peak force. This procedure assumes a linear relation between peak force and peak force variability, given that time to peak force is constant for each trial. Figure 8c demonstrates that when the individual trial peak force is multiplied by the

attained time to peak force, divided by the imposed time to peak force (200 ms), the peak force variability function becomes linear. We subsequently used this ratio technique on isometric data obtained over a broader range of times to peak force (Carlton & Newell, in press) and again found the force variability function to straighten out from the original negatively accelerating function.

It is apparent that time to peak force influences estimates of the peak force variability function. Therefore, it appears appropriate to suggest that any of the variability functions for peak force previously reported could be attained according to subject shifts in time to peak force. Thus, the square root function reported by Fullerton and Cattell (1892) is possibly due to the lengthening of time to peak force at the high force levels as demonstrated in Figure 8b from the experiment of Newell and Carlton (in press). On this basis, statistical arguments regarding the direction of error need not be invoked. Similarly, the inverted-U-function reported by Sherwood and Schmidt (1980) could be due to subjects lengthening time to peak force to such an extent that variability at high force values even decreases compared with lower peak force criteria.

To conclude, many peak force variability functions may be obtained in isometric tasks according to the limitations imposed on the time taken to reach peak force. It should be recognized that subjects modulate rate of force production and hence, time to peak force, both on a trial-by-trial basis and a group-mean basis. Because the majority of the isometric studies reviewed have shown nonproportional force variability functions, it seems appropriate to conclude that the pulse-step model (Ghez, 1979; Ghez & Vicario, 1978) of scaling-up peak force with constant time to peak is not appropriate for optimally reducing response variability.

This synthesis suggests that subjects change rate of force production according to the criterion peak force level. The rate adopted at each condition will probably be individual specific and consistent with principles of optimization in human motion. Elaborations of the force variability function for isometric tasks need to consider individual differences in force production and the impact of task constraints upon freely chosen rate of force production in assessing response variability.

*Isotonic Tasks*

In isotonic tasks the goal of the act is usually based on spatial and temporal criteria. Kinetic

parameters vary according to the kinematic constraints imposed on an individual by the task, but they rarely represent the criterion

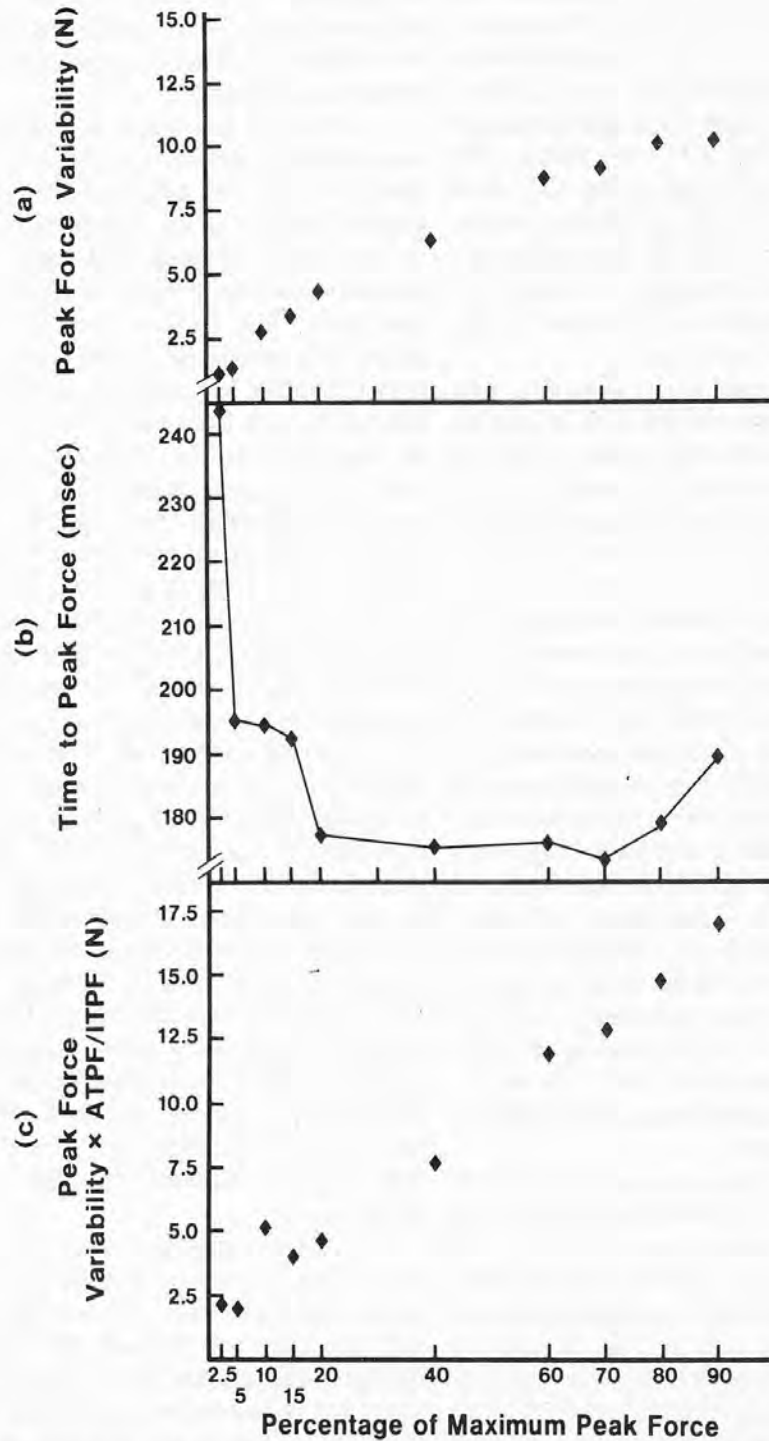


Figure 8. Group mean peak force variability as a function of % of maximum peak force (a); group mean time to peak force as a function of % of maximum peak force (b); and group mean peak force variability as a function of % of maximum peak force after each individual trial peak force was multiplied by the ratio of attained time to peak force/imposed time to peak force (c). (From "On the Relationship Between Force and Force Variability in Isometric Tasks" by K. M. Newell and L. G. Carlton, in press, *Journal of Motor Behavior*. Copyright by the Helen Dwight Reid Educational Foundation. Reprinted by permission.)

outcome. There have been fewer direct investigations of the force variability relation in movement tasks, although Newtonian mechanics dictate that this function should be congruent with the error function for the speed-accuracy relation (Hancock & Newell, in press; Schmidt et al., 1979).

Predominately, investigations in this area have concentrated on control parameters (e.g., spring stiffness, viscous damping, and inertia) that affect both the physical work required by the task and, presumably, the kinesthetic feedback associated with the response (Bahrack, 1957). Of this work, the emphasis has been on the use of spring-centered controls which allow for simultaneous presentation of distance and force cues. Because force production is proportional to movement distance in these control systems, force variability can be directly measured by distance variability. This has generally been the experimental tactic adopted. Although the confounding of distance and force parameters presents a number of interpretational problems for the description of the relation between force variability and force level, these studies provide useful insights into the parameters affecting output variability.

One of the earliest studies to examine force production in movement tasks, albeit indirectly, was that of Weiss (1954). The task of the subject was to compensate for the displacement of a spot of light from the center of an oscilloscope by moving a stick control the criterion distance and direction. A range of force-displacement conditions was used, and for all conditions the angular displacement and pressure was linearly related. Thus, although the task criterion was one of distance, this measure was proportional to force output. Weiss found that the variability of error divided through by the force of the movement decreased with distance moved, but that manipulation of pressure had no apparent effect. However, pressure and displacement were linearly related; thus, the distance manipulation is also a force manipulation. Weiss also showed a constant error shift of negative to positive overshooting with the increments of force. Our analysis of the data from this study (Weiss, 1954, Table 2) indicates that, as force level increased, the variability of force also increased at a negatively accelerating rate. Later experiments by Weiss (1955) using the same experimental protocol again suggested that dis-

tance was a more critical cue in dynamic positioning tasks.

The initial studies of Weiss suggested that force cues were not beneficial in dynamic responses, possibly because they provided only redundant information. However, a number of studies have indicated that spring-centered controls significantly improved the spatial accuracy of positioning responses (Bahrack, Bennett, & Fitts, 1955; Gibbs, 1954; Howland & Noble, 1953). Bahrack, Bennett, and Fitts' (1955) study is representative of this work. They examined the accuracy of positioning responses under constant spring tension, where tension increased with change in position of a control stick. It was proposed that these manipulations allowed an evaluation of the relative importance of cues associated with amplitude of movement per se, of terminal torque per se, and of rate of change of torque with amplitude. Bahrack and his colleagues concluded that each of these parameters influenced positioning outcome, but that positional error was smallest when the ratio of relative torque change to displacement is largest, particularly if the absolute change of torque with displacement is also large. Bahrack, Bennett, and Fitts (1955) measured performance by absolute error and, as a consequence, it is not possible to compare directly their findings with the constant and variable error functions usually displayed for an analysis of response variability. It is also surprising that in the constant torque conditions, no systematic trend occurred for variation in amplitude, as absolute amount of torque produced does vary with this manipulation.

The studies by Weiss (1954, 1955) and Bahrack, Bennett, & Fitts (1955), together with others by Bahrack, Fitts, & Schneider (1955) and Bahrack (1957), are difficult to synthesize with the isometric studies because the experimental manipulations involve a number of differences. This is because Bahrack and his colleagues were concerned principally with understanding the effect of various physical constants such as stiffness, viscous damping, and inertia upon the position, velocity, and acceleration components of movement control. The basic assumption was that the addition of control variables such as spring loading would cause a greater range of forces to be produced, leading to an increase in the number of absolutely discriminable response



categories. This effect was thought to be due to the increase in the intensity range of proprioceptive feedback associated with the response (Bahrick, 1957). These studies, therefore, might be usefully viewed as attempts to understand the contribution of torque-related parameters to response outcome rather than as descriptions of the force variability function per se. Several later studies examined the impact of the mass of the lever in discrete timing studies (e.g., Ellis, 1969; Ellis, Schmidt, & Wade, 1968), but no direct measures of force output were obtained; consequently, these studies do not contribute to the present description relating force to the variability of force.

With the recent motor-output variability model (Schmidt et al., 1979), there have been a number of studies that have explored the force variability function in isotonic responses. The basic statement of the model is that there is a proportional relation between the size of the impulse generated and the variability of the impulse. As support for the motor-output variability model, Schmidt et al. (1979) studied response variability in a variety of movement paradigms, including reciprocal and single aiming movements and rapid timing responses. As an initial test of their model, Schmidt et al. examined the variability of impulse duration when subjects made reciprocal elbow flexion-extension movements with different amplitude (16, 32, 48, and 64 degrees) and time (200, 300, 400, and 500 ms) constraints. The results showed that the within-subject variability of impulse duration increased linearly with movement time. Furthermore, there were no interactional effects between movement time and amplitude for the variability of impulse duration. These findings supported the general prediction of the motor-output variability model, whereby the variability of a given impulse parameter is proportional to the absolute level of that parameter.

As Schmidt and his colleagues (Schmidt et al., 1979) indicated, a proportional relation between the variability of impulse and impulse would lead to estimates of proportional relations between movement error and movement speed. However, there is a substantial body of data that shows that variable spatial error increases at a negatively accelerating rate for constant increments of movement speed within a given amplitude (e.g., Fitts, 1954;

Woodworth, 1899), and variable timing error decreases at a negatively accelerating rate for constant increments of amplitude within a given movement time (Newell, 1980; Newell, Carlton, Carlton, & Halbert, 1980; Newell, Hoshizaki, Carlton, & Halbert, 1979). The nonproportionality of spatial error as a function of average movement velocity can also be seen in the data presented by Schmidt and his colleagues using single aiming responses.

Figure 9 reflects an adaptation of data provided by Schmidt et al. (1979, Figures 7 and 9) that showed the relation between the effective target width ( $W_e$ ) and average movement velocity for responses of various movement times. It should be noted that the resulting effective target width represents the within-subject standard deviation of responses produced, which differs from Welford's (1968) use of the term that refers to a four-fold magnification of the standard deviation value. Figure 9 presents the  $W_e$ /distance moved (coefficient of variation) for each of the amplitude conditions at six movement times. If there is a proportional and linear relation between spatial error and average movement velocity, the relative error at each of the movement times should be equal across average velocities. Figure 9 clearly indicates that the relation between these parameters is not proportional at any of the movement times examined. Rather, there is a curvilinear relation as was shown in isometric force production tasks (e.g., Fullerton & Cattell, 1892; Jenkins, 1947a).

The motor-output variability model also predicts that when movement distance is increased in rapid timing responses with movement time held constant, the spatial errors will increase proportionally, whereas the movement timing errors are not affected (Schmidt et al., 1979). The assumption is that as the movement distance is doubled, the impulse will also double and this will cause spatial error to also double. Because in this situation the movement is being generated twice as fast at the end point of the response, twice the spatial error will be generated in the same time, leaving timing error unchanged. Also, a proportional relation would require that the addition of mass to the movement system have no effect on spatial or temporal accuracy. This is because the addition of mass has two equal and opposing influences. First, it increases the variability of the motor system due to a greater

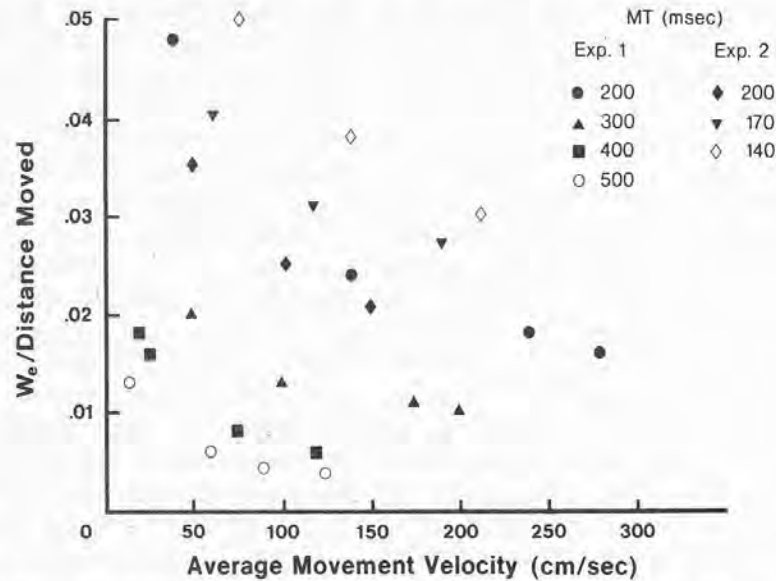


Figure 9. Coefficient of variation for aiming error as a function of movement time and average movement velocity. (From "Motor-Output Variability: A Theory for the Accuracy of Rapid Motor Acts" by R. A. Schmidt, H. N. Zelaznik, B. Hawkins, J. S. Frank, and J. T. Quinn, 1979, *Psychological Review*, 86, pp. 415-441. Copyright 1979 by the American Psychological Association. Adapted by permission of the publisher and author.)

impulse being produced. Second, it increases the inertia of the system, which provides resistance to variability in the movement (Schmidt et al., 1979). Therefore, if the relation between impulse and impulse variability is proportional, there should be no effect on movement timing accuracy when movement distance is manipulated or when the mass of the system is varied.

In a recent study (Newell, Carlton, & Carlton, 1982) using the segmental method (e.g., Clauer, McConville, & Young, 1969; Williams & Lissner, 1962), we obtained the relation between torque and torque variability from the acceleration patterns of discrete timing responses over a range of amplitudes and masses within a given movement time of 200 ms. Figure 10 shows that, consistent with our previous work (Newell, 1980; Newell et al., 1979, 1980), variable timing error decreased with increments of amplitude, and thus average velocity, and with increments of mass. Similarly, the increments in torque-related parameters (peak torque, impulse, impulse duration) were negatively accelerating functions rather than linear with gains in velocity or mass.

Newell et al. (1982) revealed that subjects lengthened time to peak force as both average velocity and mass of the system to be moved increased, despite the constant criteria move-

ment time. Thus, as in isometric tasks, subjects do not simply scale-up peak force with impulse timing parameters remaining constant. Generally, adding mass reduces the rate of acceleration because subjects lengthen the time to peak force.

Danoff (1978) has shown similar impulse relations in a maximal velocity elbow flexion task where subjects lengthen the time to peak force as load on the limb is increased (see Figure 11). This modulation of rate of force production therefore not only decreases movement velocity in maximal velocity tasks, but importantly, could reduce the variability in accuracy tasks. Indeed, variations in rate may explain the reduction in movement variability error found by Schmidt and Sherwood (1982) at very high force levels.

Taken collectively, the studies for isotonic tasks do not present such a coherent picture as the isometric tasks for the force variability function. Part of this obfuscation is due to the range of manipulations that have occurred, including spring, damping, and inertial changes. However, in conditions that are unchanging throughout the movement, the curvilinear function depicted in Newell et al. (1982) is compatible with that previously shown for isometric tasks and the kinematic analysis of response variability (Hancock &

Newell, in press). We now study a projection of the force variability function that is consistent for both isometric and isotonic tasks.

Projected Force Variability Function

In both isometric and isotonic tasks, the performer exerts force over time. Isometric tasks do not allow the performer to overcome the inertia of an object and, as a consequence, neither movement nor length changes in muscle occur. Despite these differences, both tasks require the performer to generate an impulse, which will vary systematically according to its relative size for the muscle group engaged in the action. Consequently, comparisons of variability need to be made on the basis of the percentage of maximum output for any

given muscle group. Even within this framework, inconsistencies might arise due to the different structure of muscle groups and the synergistic organization of the limb links involved. However, this level of analysis and its possible significance for force variability functions is beyond the scope of the present article.

The projected functions apply to both peak force and impulse measures. As with the speed-accuracy relation, we propose that an analysis of the first four distributional moments is required to understand fully force production characteristics over different conditions (Newell & Hancock, in press). We begin by considering the constant error function which only emerges when a force parameter is established as a criterion of the task.

Figure 12a displays the constant error func-

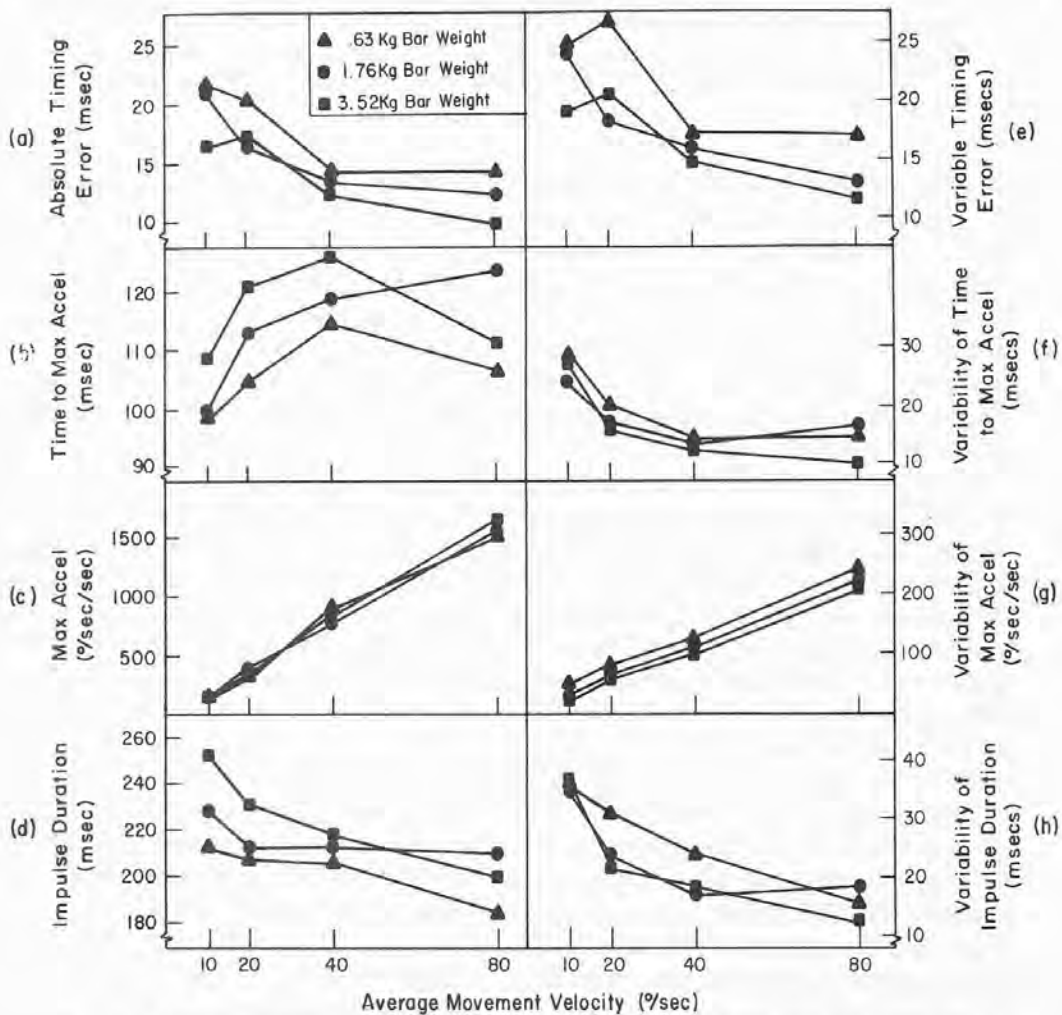


Figure 10. Movement-related parameters as a function of average velocity and mass of the system moved. (From "The Relationship of Impulse to Timing Error" by K. M. Newell, L. G. Carlton, and M. J. Carlton, 1982, *Journal of Motor Behavior*, 14, pp. 24-45. Copyright 1982 by the Helen Dwight Reid Educational Foundation. Adapted by permission.)



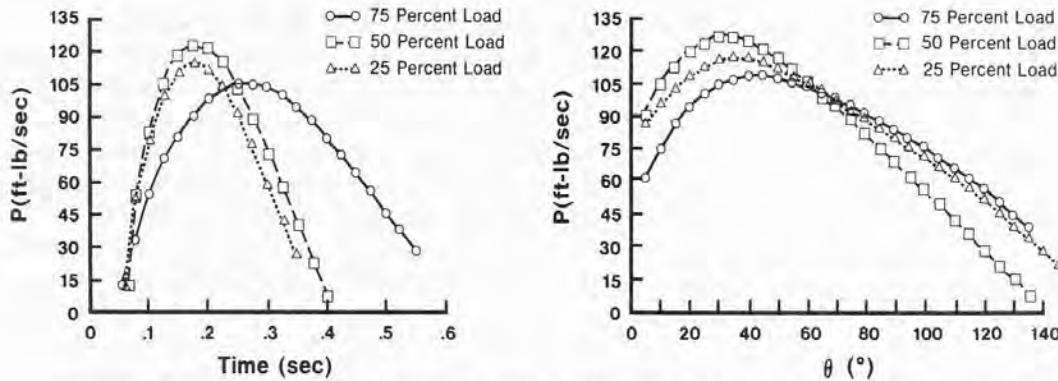


Figure 11. Power versus time, and power versus angular displacement as a function of limb load. (Reprinted with permission from *Journal of Biomechanics*, 11, J. V. Danoff, "Power Produced by Maximal Velocity Elbow Flexion." Copyright 1978, Pergamon Press, Ltd.)

tion over the range of zero to maximum force for any particular muscle group. At low force levels overshooting occurs, which decreases to zero and subsequently increases in the degree of undershooting as the absolute force level increases toward maximum for any prescribed

action. It is not certain at what percentage of maximum the change from overshooting to undershooting occurs. We have projected this at approximately 50% of maximum, although there is no apparent a priori reason why this is the case. The range effect appears to be determined by the order of presentation of conditions and the magnitude of the response conditions presented to the subject (Ellson & Wheeler, 1949).

The relation between absolute level of force which the subject is constrained to produce and the variability of repeated response is displayed in Figure 12b. It depicts variability, as measured by within-subject standard deviation, as increasing at a negatively accelerating rate for equal increments of absolute force production. The curve is of an exponential type, although there are insufficient data sets to determine its exact form and hence its compatibility to the square root function originally proposed by Fullerton and Cattell (1892). The goal of defining a single force variability function independent of other impulse parameters becomes unnecessary, however, given that the function is facile to modulations in the way that the subject produces the impulse in response production. The variability function is plotted against the imposed force criterion rather than the attained force value, so that the abscissae for Figure 12a and 12b are identical.

It is important to note that the constant and variable error functions are posited to pass through the origin. Zero error occurs when no force is generated, although it is recognized that if a measurement was made in this sit-

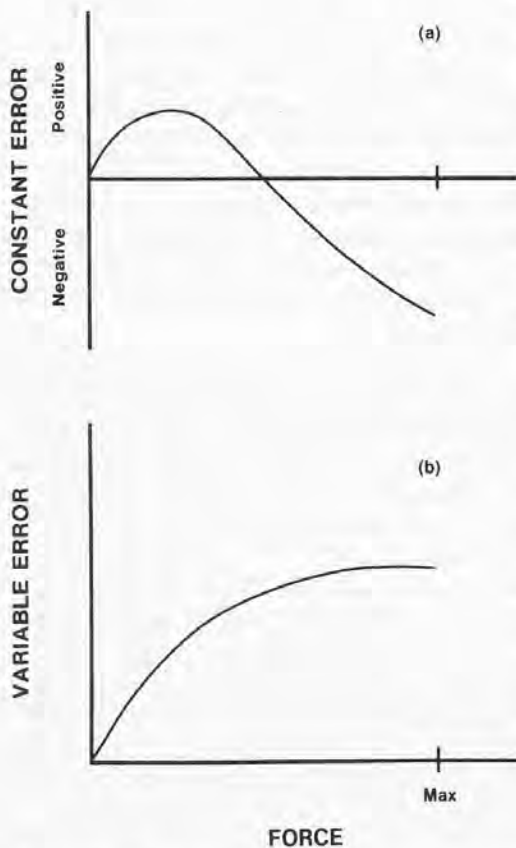


Figure 12. Projected functions for force parameters over the range of force production for both isometric and isotonic tasks: (a) constant error, (b) variability.

uation, a measurement error could occur that would have the artifactual effect of creating an intercept in practice. Thus, we have imposed the important theoretical constraint of the functions passing through the origin.

Following Fullerton and Cattell (1892), we anticipate that the actual force distributions will deviate from normality in a systematic fashion, with shifts in the percentage of maximum for the absolute level of force being exerted. At low force levels, the distributions will be slightly skewed in a positive direction and have a high degree of leptokurticness. As force level increases, these deviations will dissipate and at 50% of maximum force the distribution will approximate normality. Beyond this point the complementary bias will exist with increasing forms of high negative skewness and a modicum of platykurticness. These biases in the third and fourth moments are essential to capture because they influence the way in which interpretations of the change in the first and second moments are made. Similarly, interpretations of the variability of force can only be made in view of an understanding of the constant error shift. In summary, the first four moments are required to understand fully response variability (Newell & Hancock, in press).

It is important to recognize that the projected functions for force variability are based upon what subjects typically produce in force production tasks, not what they can produce. As indicated in the isometric synthesis, a variety of force variability functions probably can be attained. It appears that task constraints dictate the force variability function that will be generated because, as evidenced in isometric tasks, subjects will systematically modulate the manner in which force is produced as a function of the force requirements of the task. These adaptations by the subject to task demands have a number of implications for models of force production, and we close by assessing these in view of the derived force variability function.

#### Force Variability and Models of Response Production

The present synthesis of the relation between force and force variability in both isometric and isotonic tasks suggests that a negatively

accelerating rate of force variability is produced when equal increments of force are manipulated across the continuum available to the performer. This variability function for force appears consistent with the space-time analysis of the movement speed-accuracy relation previously outlined (Hancock & Newell, in press), although we have yet to advance a formal link between the kinematic and kinetic analyses of response variability. Schmidt et al. (1979) outlined such a link, but the internal validity of the predictions for force, space, and time response parameters is inconsistent (Hancock & Newell, in press). However, the motor-output variability model (Schmidt et al., 1979) did attempt an explicit link between the kinematic and kinetic accounts of response variability, a relation that was only implicit in the early work of Fullerton and Cattell (1892).

The force variability studies reviewed do not provide direct tests of the competing accounts of the movement speed-accuracy relation (e.g., Fitts, 1954; Keele, 1968; Schmidt et al., 1979). In particular, they do not provide a basis to understand the role of feedback in force production, because insufficient time durations of force production have been manipulated. However, the movement speed-accuracy data suggest that the availability of vision does not change the function for response variability; rather, it only changes a constant in the function relating to the absolute level of movement accuracy (Hancock & Newell, in press).

The current kinetic analysis of response variability reveals several limitations to extant models of force production. Subjects apparently attempt to minimize response variability by modulating the rate of force production for a given set of peak force (isometric) or kinematic (isotonic) task constraints. In isometric tasks, subjects do not scale-up peak force by holding time to peak force constant as suggested explicitly by the pulse-step (Ghez & Vicario, 1978) and implicitly by motor-output variability (Schmidt et al., 1979) models. Rather, they systematically vary the time to peak force according to task constraints as they do in isotonic tasks (Danoff, 1978; Newell et al., 1982). The basis for this variation in the rate of force production is not elaborated in this synthesis of the force variability relationship, but it appears consistent with principles

of efficiency in muscular contraction, where efficiency of human motion is defined by the ratio of work done to energy expended.

In his pioneering studies of the physiology of muscular work, Hill (1922) investigated the speed of muscular contraction and its relation to mechanical efficiency in human skeletal muscle. He proposed that rate of contraction is the key parameter in determining the mechanical efficiency of muscle. Furthermore, there is a nonlinear relation between efficiency losses in muscular contraction and the deviation from the optimal contraction duration. Efficiency has subsequently been invoked as an emergent property in theorizing regarding the coordination and control of movement (c.f., Nelson, 1983; Sparrow, 1983), but a formal link of response variability and efficiency criteria has not been advanced.

It is conjectured that, after practice, an individual's freely chosen rate of force production is optimal for the efficiency of muscular contraction (Cavagna, 1977; Sparrow, 1983). In this regard it is interesting to note that modulations in rate of force production with increments of peak force appear more dramatic and systematic when repeated trials at a given set of task constraints are blocked (e.g., Freund & Budingen, 1978; Newell & Carlton, *in press*) than when the task criteria vary from trial to trial (Ghez & Vicario, 1978). Presumably, it is only the former situation that allows the optimal rate of force production to be approached, at least within the practice typically given in force production studies. Thus, even the task constraint of blocked versus unblocked force level criteria may influence the emergent force variability function-by way of differing subject strategies to generate the impulse.

Efficiency is an attractive hypothesis to pursue, not only because of the theoretical basis provided by Hill (1922) and subsequent analyses of the efficiency of muscle mechanics, but also because it can encapsulate arguments that may be advanced regarding the role of specific physiological mechanisms, such as motor unit recruitment, in the variability function (Hatze & Buys, 1977). Efficiency principles also reaffirm the view that the coordination and control of human movement cannot be understood purely from mechanical principles alone. Although this position has gained in acceptance by those developing biomechanical op-

timization techniques (e.g., Chow & Jacobson, 1971), it is evident that the pulse-step (Ghez & Vicario, 1978) and the motor-output variability (Schmidt et al., 1979) models are founded upon a Newtonian model of physical systems. Models from nonbiological systems can represent useful analogies for guiding the development of theorizing in motor control. If the behavior itself is to be modeled, however, biological constraints need to be introduced into physical system models of response variability.

Efficiency of muscle mechanics is consistent with a peripheral hypothesis regarding response variability but central mechanisms can also contribute to performance variability in force production. One premise of the motor-output variability model is that repeated responses enable the same motor program, and thus variability from central mechanisms is minimized experimentally (Schmidt et al., 1979). Therefore, within this approach, it is noise in the neuromuscular system, inherent in the repeated execution of a specific motor program, that is responsible for response variability rather than intrinsic variation in the program itself. Timing models of repetitive or sequential responses (e.g., Wing & Kristofferson, 1973) have attempted to partition the contribution of central and peripheral mechanisms to response variability, but a similar distinction has not been adopted with discrete actions.

Our synthesis suggests that central mechanisms may well contribute to the response variability function. This requires a broadening of the theoretical backdrop to response variability beyond peripheral muscular considerations. Whether central mechanisms can also be related to efficiency in the variability of discrete responses, as it can to the gait pattern shifts of locomotory activities (e.g., Bolis, Schmidt-Nielson, & Maddrell, 1973), remains to be determined. A more general and significant feature to the efficiency concept is that it may stretch beyond being a consequence of muscular organization. Efficiency could be a principle that specifies a priori a particular biokinematic organization of the organism (Kugler, Kelso, & Turvey, 1980; Sparrow, 1983), and constrains the interaction of central and peripheral contributions in the coordination and control of movement. The impli-



cation of an efficiency orientation to motor control is that response variability will reflect the degree to which task and environmental constraints demand deviations from organically optimal kinematics and kinetics.

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